Introduction to Sage

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Sage Days 86
Université du Québec à Montréal
April 18th 2017

This week outline

Four invited talks during the week :

- Tuesday : Introduction
- Wednesday : James Mitchell and François Bergeron
- Thursday : Thierry Monteil
- Friday : Jennifer S. Balakrishnan

The rest of the time, there will be guided tutorials and lot of free time to work and discuss.
During this free time, it is expected that :

- You do tutorials/exercises on theme you decide.
- You work on your project.
- You ask questions.

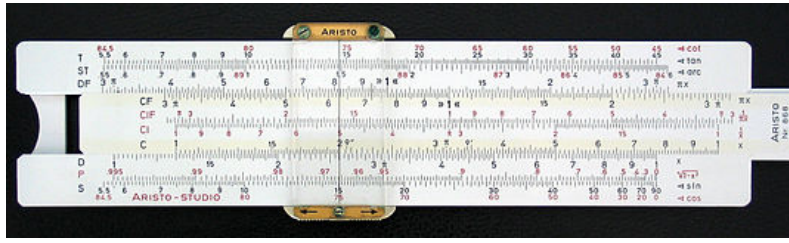
Outline

Plan

From Abacus to computers

Abacus (2000 BC)

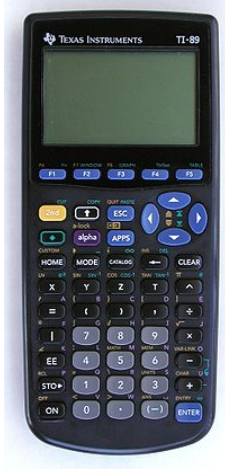
Pascaline (1645)



Slide rule (1620)

HP-35 (1972)

IMB PC 5150 (1983)



TI-89 (1998)

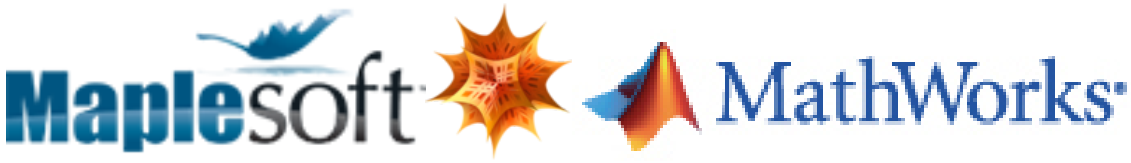
Depuis 1960, au moins 45 logiciels de mathématiques
(32 à licence libre et 13 commerciaux) :

Axiom	FORM	Magnus	MuPAD	SyMAT
Cadabra	FriCAS	Maple	OpenAxiom	SymbolicC
Calcinator	FxSolver	Mathcad	PARI/GP	Symbolism
CoCoA-4	GAP	Mathematica	Reduce	Symengine
CoCoA-5	GiNaC	MathHandbook	Scilab	SymPy
Derive	KANT/KASH	Mathics	SageMath	TI-Nspire
DataMelt	Macaulay2	Mathomatic	SINGULAR	Wolfram A
Erable	Macsyma	Maxima	SMath	Xcas/Giac
Fermat	Magma	MuMATH	Symbolic	Yacas

Source : `http://en.wikipedia.org/wiki/List_of_computer_algebra_systems`

Quelques logiciels commerciaux :

- Maple, Waterloo Maple Inc., Maplesoft, since 1985.
- Mathematica, Wolfram Research, since 1988.
- Matlab, MathWorks, since 1989
- Magma, University of Sydney, since 1990



MAGMA
COMPUTER ALGEBRA

Quelques logiciels libres :

- Maxima, W. Schelter et coll., since 1967 : symbolic operations
- Singular, U. of Kaiserslautern, since 1984 : polynomial comput.
- PARI/GP, U. Bordeaux 1, since 1985 : number theory computations
- GAP, GAP Group, since 1986 : computational group theory
- R, U. of Auckland, New Zealand, since 1993 : statistiques

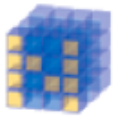
"You can read Sylow's Theorem and its proof in Huppert's book in the library . . .then you can use Sylow's Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly . . ..

With this situation two of the most basic rules of conduct in mathematics are violated : In mathematics information is passed on free of charge and everything is laid open for checking."

—J. Neubüser (1993)

(il a fondé GAP en 1986)

Python scientifique



NumPy
Base N-dimensional array package

SciPy library
Fundamental library for scientific computing

Matplotlib
Comprehensive 2D Plotting

jupyter

IP[y]: IPython

IPython
Enhanced Interactive Console

Sympy
Symbolic mathematics

pandas
Data structures & analysis

sage

- 1991 : première version de Python
- 2000-2001 : Matplotlib, IPython, SciPy
- 2006-2008 : NumPy, SymPy, pandas
- 2006 : Sage based on PARI, Maxima, Python, Singular, GAP.
- 2012-2014 : Julia, Jupyter
- 2015 : 70 000 librairies Python dans le *Python Package Index*

Plan

Mission

The Sage Project aims to create a viable high-quality and *open-source* alternative to Magma, Maple, Mathematica, Matlab and MuPAD, and to foster a friendly community of users and developers.

Sage is an *open source* software

Sage is distributed under the terms of the GNU General Public License version 2 (GPLv2) which guarentees four types of freedom :

- The freedom to use the software (it is free).
- The freedom to read the source code.
- The freedom to improve the software.
- The freedom to redistribute the modified software to anyone.

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Sage is . . .

a *distribution* of softwares

Sage is a free software : mutualization

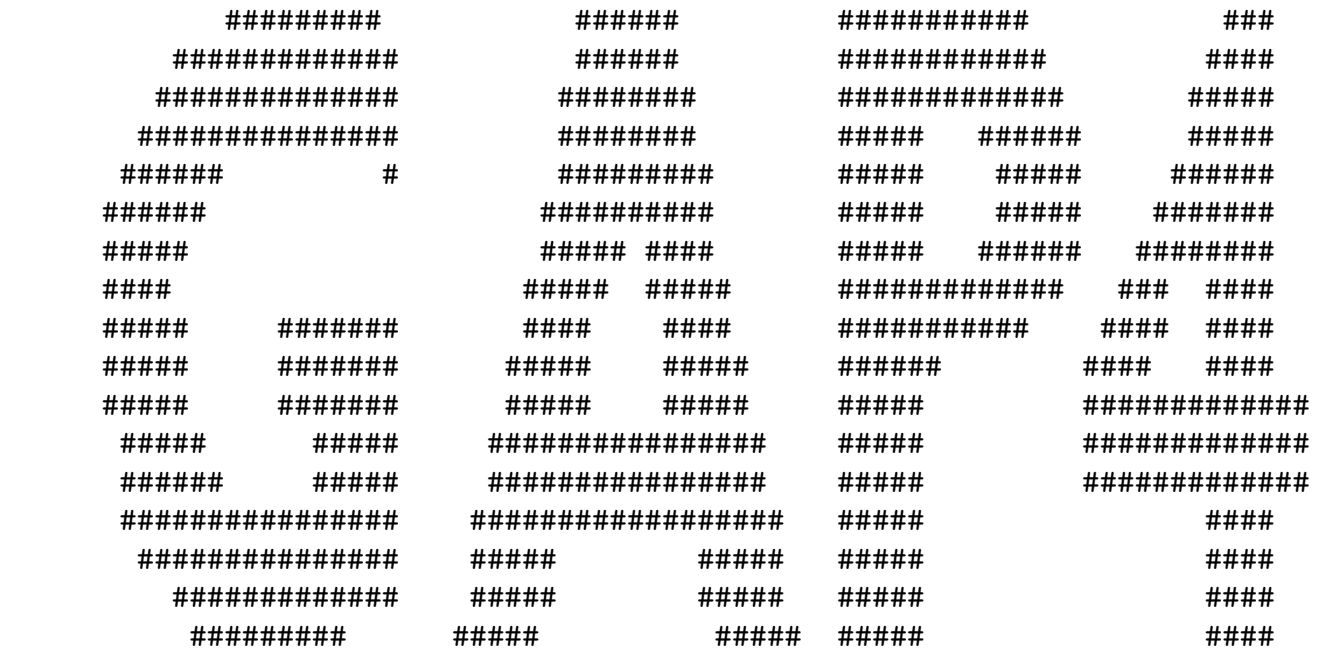
Build the bike instead of reinventing the wheel :

Arbitrary precision arithmetic	MPIR (GMP), MPFR, MPFI, NTL
Algebra	GAP, Maxima, Singular, Givaro
Algebraic geometry	Singular, Macaulay2*
Arithmetic geometry	FLINT, PARI/GP, NTL, ecm
Courbes elliptiques et fonctions L	ECLib, mwrank, ratpoints, SYMPOW, Lca
Symbolic computation	Pynac, Maxima, Sympy, giac*
Exact linear algebra	Linbox, IML
Numerical calculations	Blas (Atlas), Numpy, LAPACK
Numerical calculations	GSL, Scipy
Combinatorics	Symmetrica, Lrcalc, PALP, Coxeter 3, Che
Graph theory	NetworkX, Cliquer, Buckygen*, graphviz*, cvxopt, PPL, glpk, CBC*
Group theory	GAP
Game theory	Gambit*
Statistics	R, Rpy, pandas*
Cryprography	pycrypto, cryptominisat*

(* optional)

. . . and more !

```
> sage -gap
```



```
    Information at:  http://www.gap-system.org
    Try '?help' for help. See also  '?copyright' and  '?authors'

   Loading the library. Please be patient, this may take a while.
GAP4, Version: 4.4.10 of 02-Oct-2007, x86_64-unknown-linux-gnu-gcc

gap>
```

```
> sage -singular

                     SINGULAR                                 /  Development
 A Computer Algebra System for Polynomial Computations       /   version 3-1-0
                                                            0<
     by: G.-M. Greuel, G. Pfister, H. Schoenemann             \   Mar 2009
FB Mathematik der Universitaet, D-67653 Kaiserslautern         \
>
```

```
> sage -maxima

Maxima 5.16.3 http://maxima.sourceforge.net
Using Lisp ECL 9.4.1
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1)
```

```
> sage -gp

                GP/PARI CALCULATOR Version 2.3.3 (released)
          amd64 running linux (x86-64/GMP-4.2.1 kernel) 64-bit version
           compiled: Jul 10 2009, gcc-4.3.2 (Ubuntu 4.3.2-1ubuntu12)
                (readline v5.2 enabled, extended help available)

                    Copyright (C) 2000-2006 The PARI Group

PARI/GP is free software, covered by the GNU General Public License, and
comes WITHOUT ANY WARRANTY WHATSOEVER.

Type ? for help, \q to quit.
Type ?12 for how to get moral (and possibly technical) support.

parisize = 8000000, primelimit = 500000
?
```

```
> sage -R

R version 2.6.1 (2007-11-26)
Copyright (C) 2007 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

>
```

Sage *combines* software.

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[This examples are from a presention of William Stein]

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Construct an elliptic curve using *John Cremona's table* :

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sage: E = EllipticCurve('389a')
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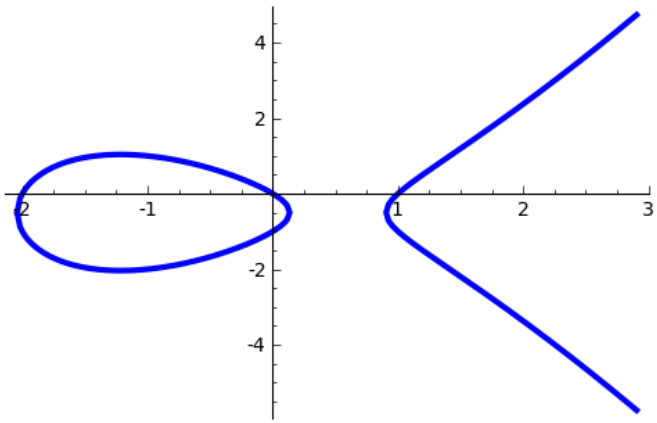
sage: plot(E,thickness=3)
```



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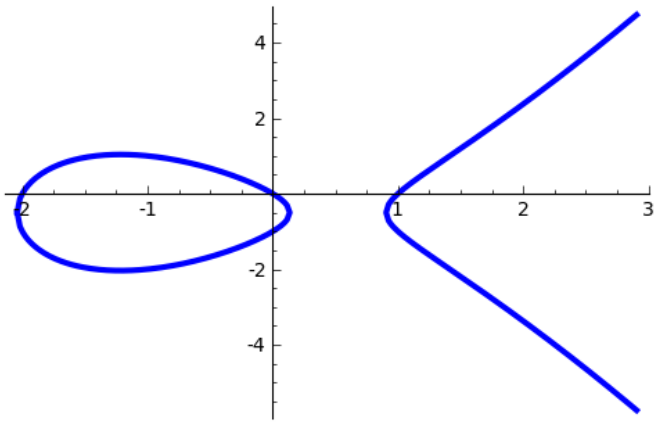

mwrank to do a 2-descent :

```
sage: E.mwrank()
Curve [0,1,1,-2,0] :     Rank = 2
```

Sage *combines* software

PARI to compute Fourier coefficients a_n :

```
sage: E.anlist(15)
[0, 1, -2, -2, 2, -3, 4, -5, 0, 1, 6, -4, -4, -3, 10, 6
```

Sage *combines* software

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```

lcalc to compute zeros in the critical strip of the L-series :

```
sage: E.lseries().zeros(5)
[0.000000000, 0.000000000, 2.87609907, 4.41689608, 5.79
```

Sage *combines* software

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sympow to compute the modular degree :

```
sage: E.modular_degree()
40
```

Sage *combines* software

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sympow to compute the modular degree :

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40
```

Magma to compute the rank of the 3-selmer group :

```
sage: magma(E).ThreeSelmerGroup()
Abelian Group isomorphic to Z/3 + Z/3
Defined on 2 generators
```

Sage *combines* software

Let's integrate $\int \cos(x^2)dx$:

```
sage: integrate(cos(x^2), x)
-1/8*((I + 1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) +
(I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x))*sqrt(pi)
```

Software used for this computation :

```
sage: from sage.misc.citation import get_systems
sage: get_systems("integrate(cos(x^2), x)")
['MPFI', 'ginac', 'GMP', 'Maxima']
```

Sage uses *Python*
as its programming language.

Sage uses *Python*

- Sage $\approx$ Python + a huge Python library
- Sage may be the first successful math software system to not invent its own new language just for mathematics.
- Tens of thousands of third party Python packages are immediately available for use with Sage !
- Easy to write and read :

math : $\left\{ 17x \middle| x \in \{0, 1, ..., 9\} \text{ et } x \text{ est impair} \right\}$

python :
```
[17*x for x in range(10) if x % 2 == 1]
```

Sage Important Philosophy : Elements have parent

```
sage: m = matrix(ZZ, 3, [2,3,4,2,4,6,8,8,5])
sage: m.parent()
Full MatrixSpace of 3 by 3 dense matrices over Integer Ring
sage: m.echelon_form()
[2 0 1]
[0 1 2]
[0 0 3]

sage: m = matrix(QQ, 3, [2,3,4,2,4,6,8,8,5])
sage: m.parent()
Full MatrixSpace of 3 by 3 dense matrices over Rational Fiel
sage: m.echelon_form()
[1 0 0]
[0 1 0]
[0 0 1]
```

Sage Optional Packages (spkg)

http://www.sagemath.org/packages/optional/

Fokko Ducloux's Coxeter 3 C++ library

```
sage -i coxeter3-1.1.spkg
```

Gnuplot.py is a Python package that interfaces to gnuplot

```
sage -i gnuplotpy-1.8.spkg
```

Ore Polynomials in Sage

http://www.risc.jku.at/research/combinat/software/ore_algebra/

```
sage -i ore_algebra-0.1.spkg
sage: from ore_algebra import *
```

SageManifolds : Differential geometry and tensor calculus

http://sagemanifolds.obspm.fr/

```
sage -i http://sagemanifolds.obspm.fr/spkg/manifolds-0.2.spkg
sage: from manifolds.all import *
```

Plan

Community

You are not alone !

Working together with internet

Development :

- `http://groups.google.com/group/sage-devel`
- `http://groups.google.com/group/sage-release`
- `http://trac.sagemath.org/sage_trac/`

Support :

- `http://groups.google.com/group/sage-support`
- `http://ask.sagemath.org/questions/`
- `http://wiki.sagemath.org/`

irc-channel

- `#sage-devel` on `freenode.net`

Developer Map

There are currently $244 + 1 + 4 + 11 + 9$ contributors in $165 + 1 + 2 + 9 + 5$ different places from all around the world.

`http://www.sagemath.org/development-map.html`

Sage Days

Sage Days are workshops aiming at

- fix bugs and develop new functionalities
- introduce new users and developers

Dozen of workshops are organized every year all around the world.

Sage Days in 2010

`https://wiki.sagemath.org/Workshops`

- Sage Days 19 : Seattle, USA (January 2010)
- Sage Days 20 : Marseille, France (February 2010)
- Sage Days 20.25 : Montreal, Canada (March 2010)
- Sage Days 20.5 : Toronto, Canada (May 2010)
- Sage Days 21 : Seattle, USA (June 2010)
- Sage-Combinat/Chevie : France (June 2010)
- Sage Days 22 : Berkeley, USA (July 2010)
- Sage Days 23 : Leiden, Netherlands (July 2010)
- Sage Days 23.5 : Kaiserslautern, Germany (July 2010)
- Sage Days 24 : Linz, Austria (July 2010)
- Sage Days 25 : Mumbai, India (August 2010)
- Sage Days 25.5 : Montreal, Canada (September 2010)
- Sage Days 26 : Seattle, USA (December 2010)

Plan

Some functionalities

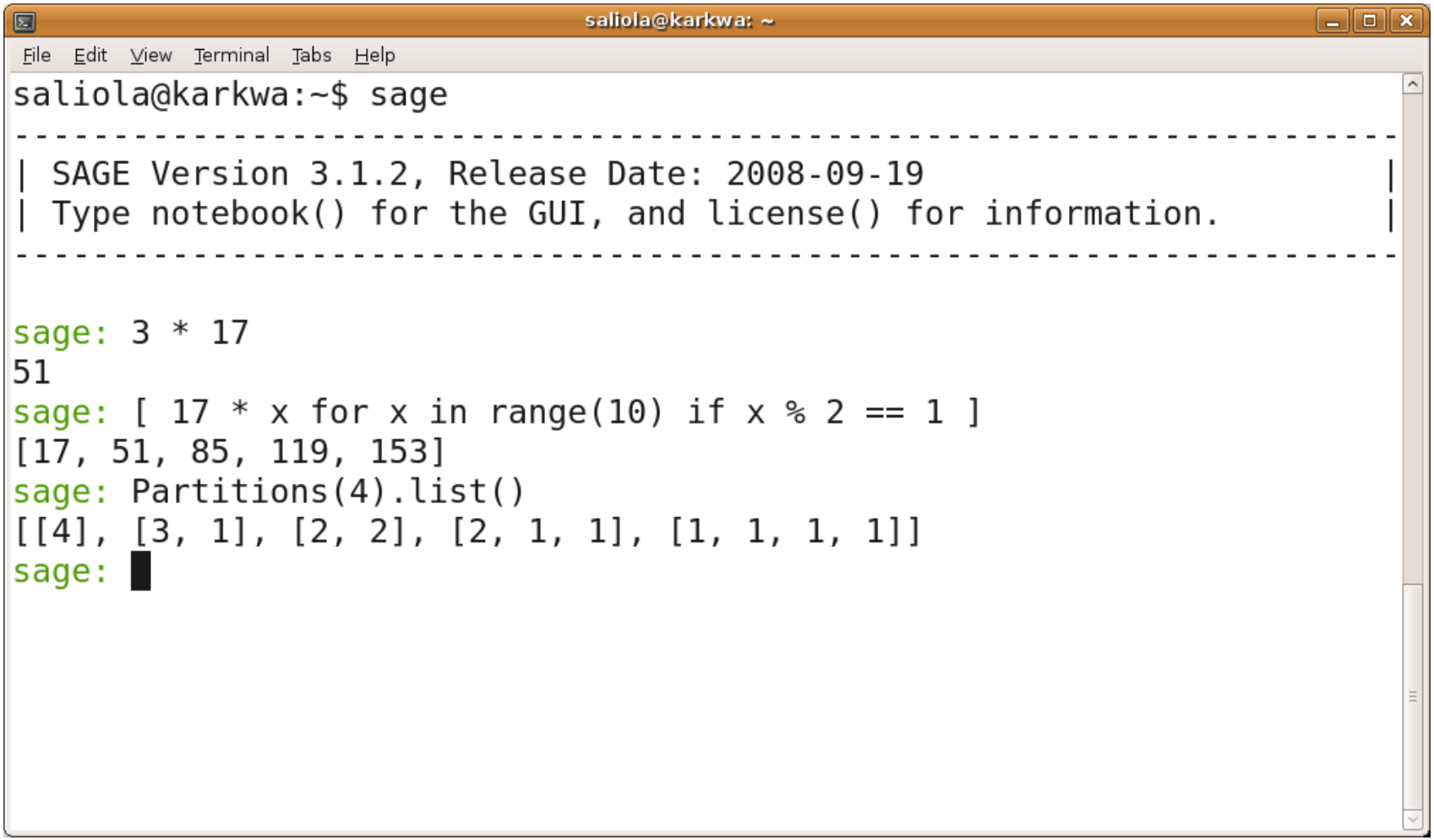
Interfaces :

- Command line
- The old Sage or the new Jupyter Notebook (local)
- Sage Cell `http://sagecell.sagemath.org/`
- Sage Eval `http://www.sagemath.org/eval.html`
- Sage Cloud (internet)

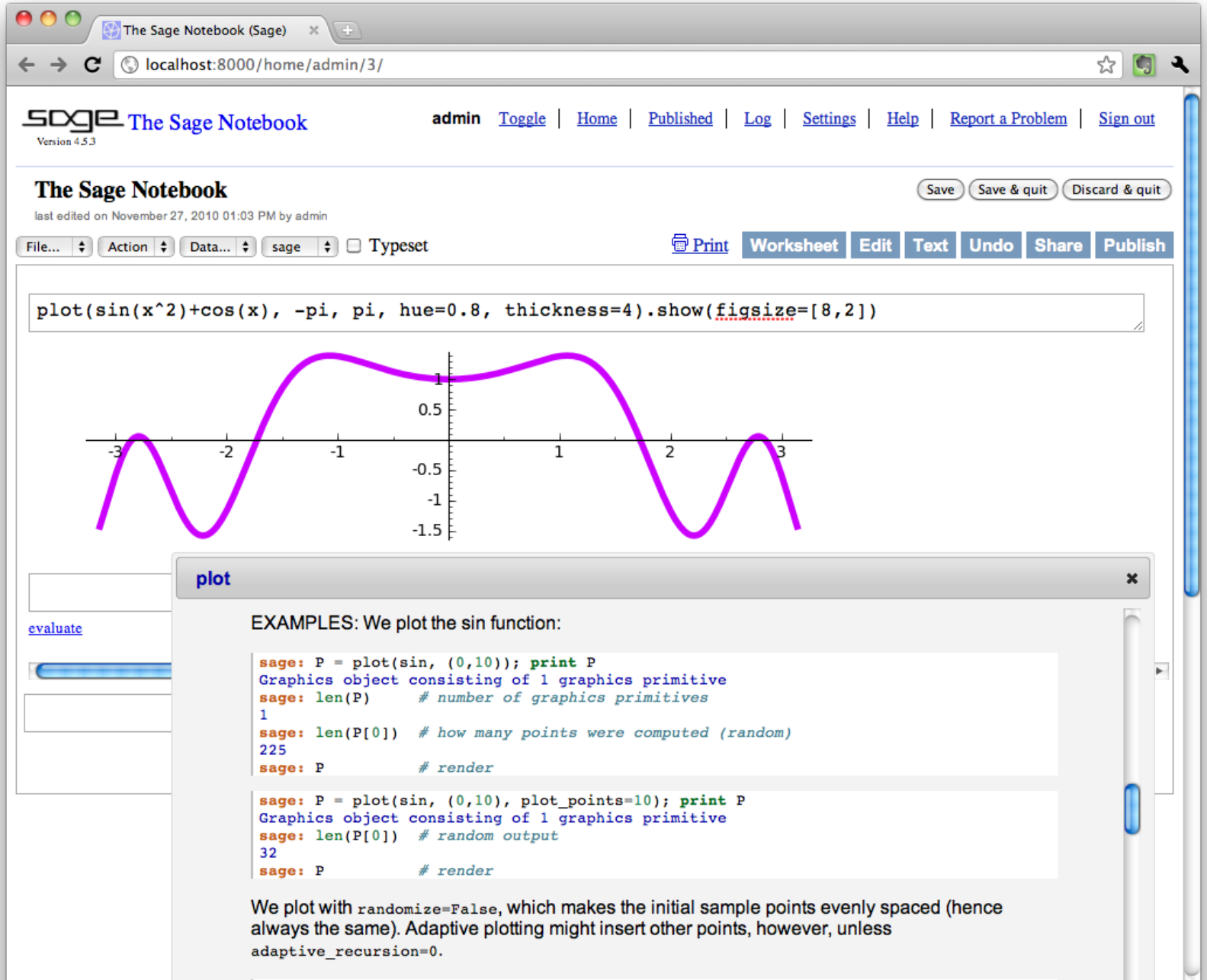
Interesting features :

- Sagetex (Dan Drake, Corée du Sud)
- Interact and animations
- Cython : translates Python code $\mapsto$ C code.

Command line interface

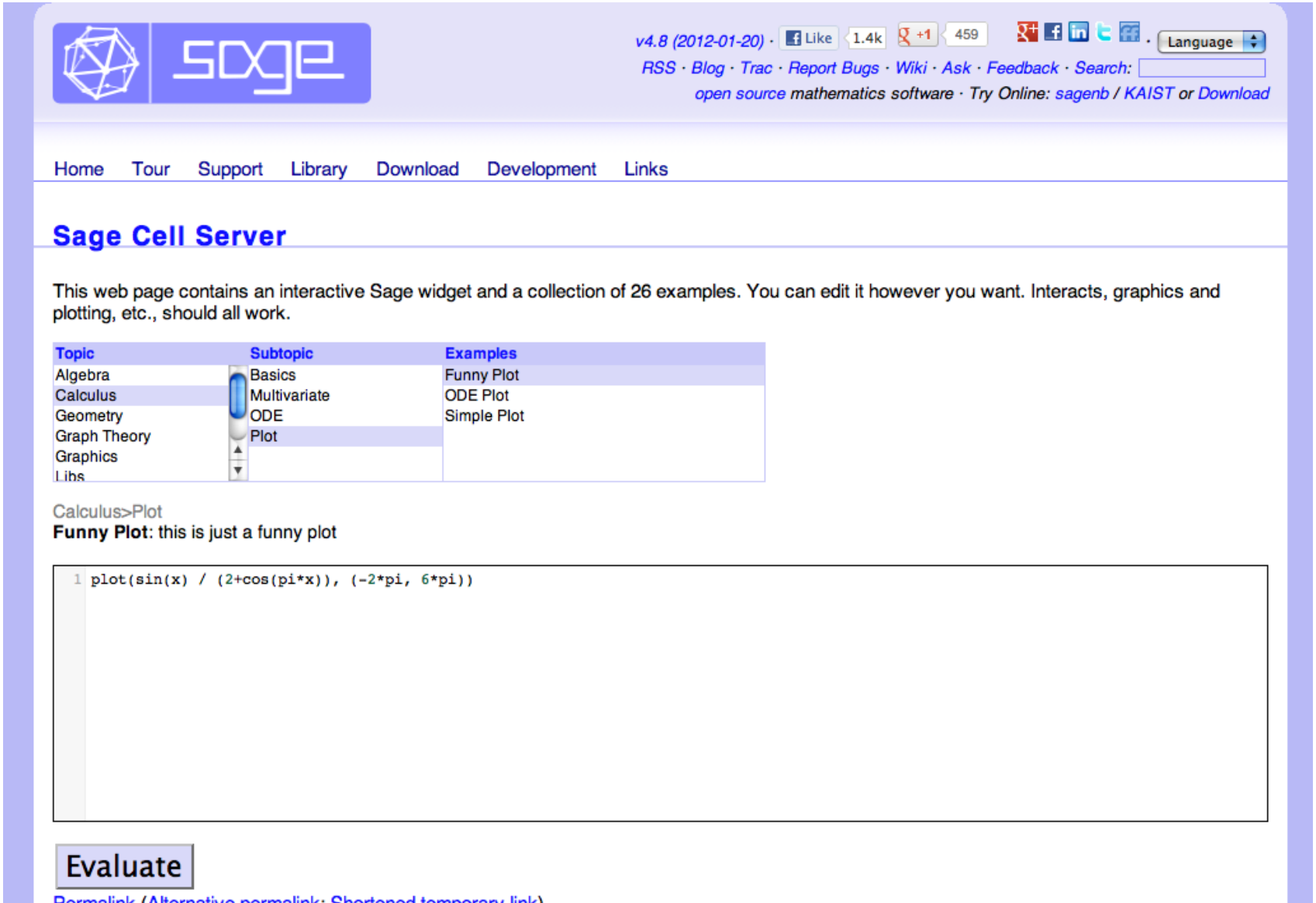


Notebook interface (local)



Sage Cell Server

http://www.sagemath.org/eval.html



LaTeX

In this LaTeXfile, I write

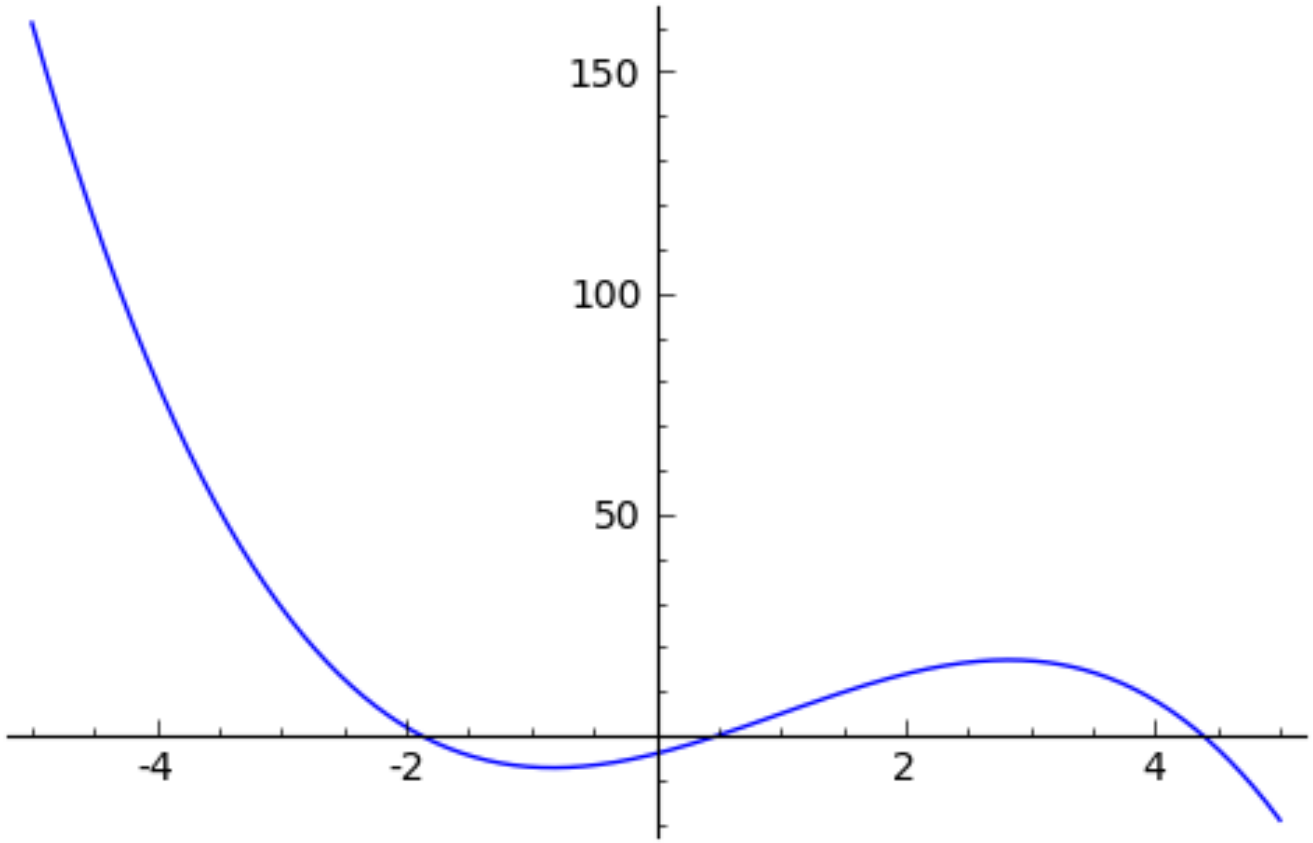
```
\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}
```

LaTeX

In this LaTeXfile, I write

```
\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}
```

and this is replaced by :



LaTeX

In this LaTeXfile :

```
\begin{sagesilent}
  sigma = Permutation([7,3,1,5,2,6,8,4])
  P, Q = sigma.robinson_schensted()
\end{sagesilent}

Let $\sigma = \sage{sigma}$. The
Robinson-Schensted-Knuth algorithm produces
the tableaux:
    \[\sage{P} \quad \sage{Q}\]
```

LaTeX

It got replaced with :

Let $\sigma =$ `[7, 3, 1, 5, 2, 6, 8, 4]`. The Robinson-Schensted-Knuth algorithm produces the tableaux :

1	2	4	8
3	5	6	
7			

1	4	6	7
2	5	8	
3			

LaTeX

It got replaced with :

Let $\sigma =$ [7, 3, 1, 5, 2, 6, 8, 4]. The Robinson-Schensted-Knuth algorithm produces the tableaux :

1	2	4	8
3	5	6	
7			

1	4	6	7
2	5	8	
3			

This is done with the sagetex package for LaTeX, written by Dan Drake. Of course, the package is included with Sage.

For more information

- http://sagemath.org/help.html
- http://sagemath.org/doc
- http://ask.sagemath.org/questions/
- http://wiki.sagemath.org

Ressources in French :

- http://sagemath.org/fr
- http://sagemath.org/fr/html/tutorial

Le livre Calcul mathématique avec Sage :

- http://sagebook.gforge.inria.fr/