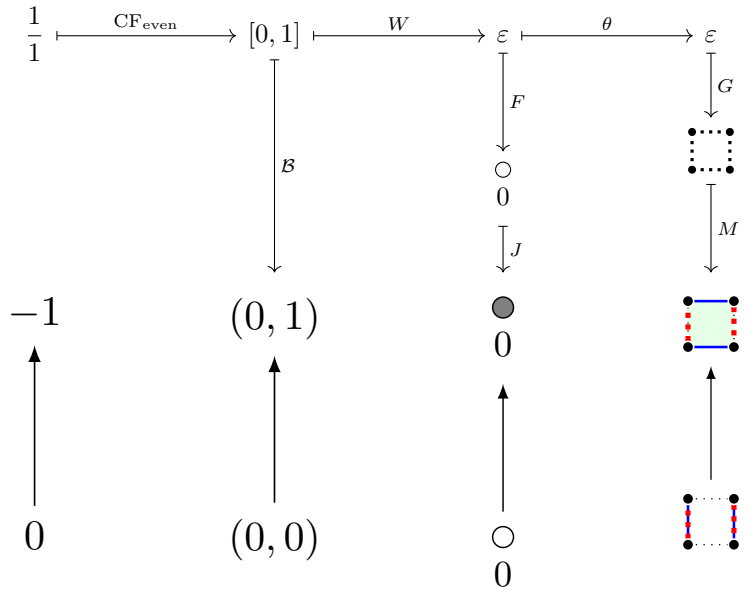
$\frac{1}{1}$
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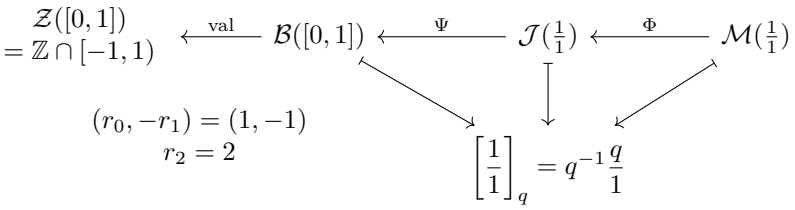
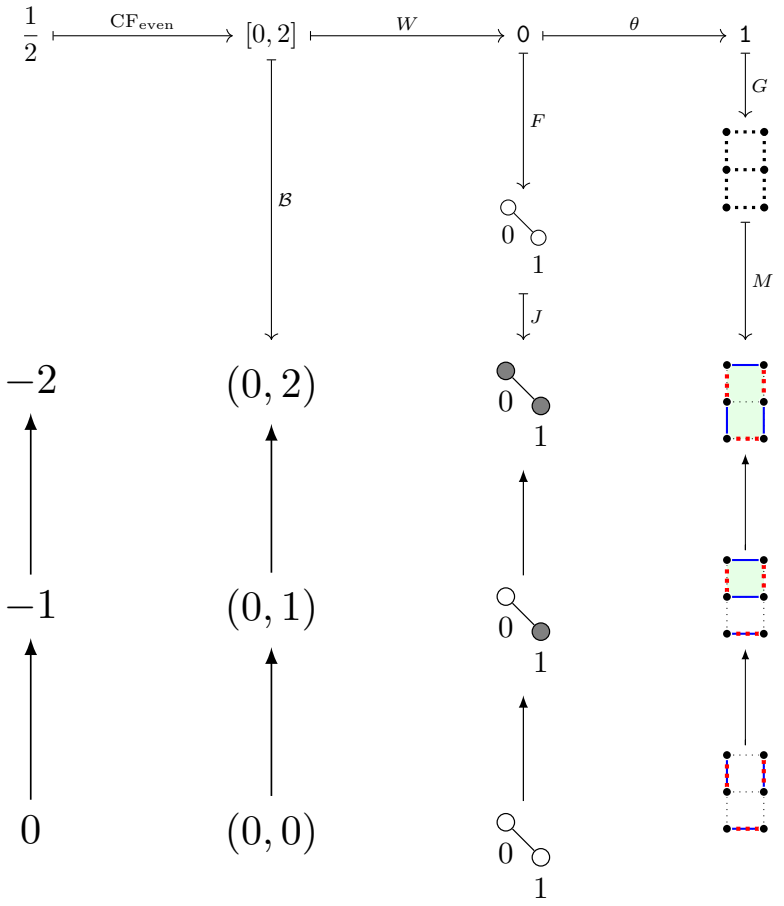

$\mathcal{Z}([0,1])$
$= \mathbb{Z} \cap [-1,1)$
val
$\mathcal{B}([0,1])$
Ψ
$\mathcal{J}(\frac{1}{1})$
Φ
$\mathcal{M}(\frac{1}{1})$
$(r_0, -r_1) = (1,-1)$
$r_2 = 2$
$\left[\frac{1}{1}\right]_q = q^{-1}\frac{q}{1}$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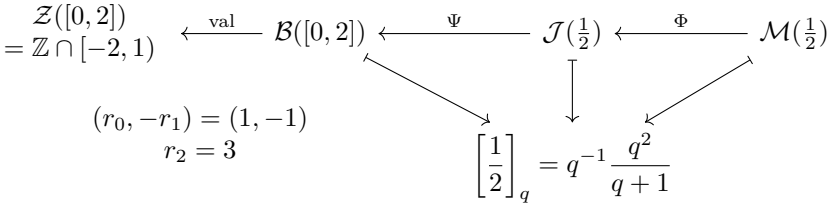
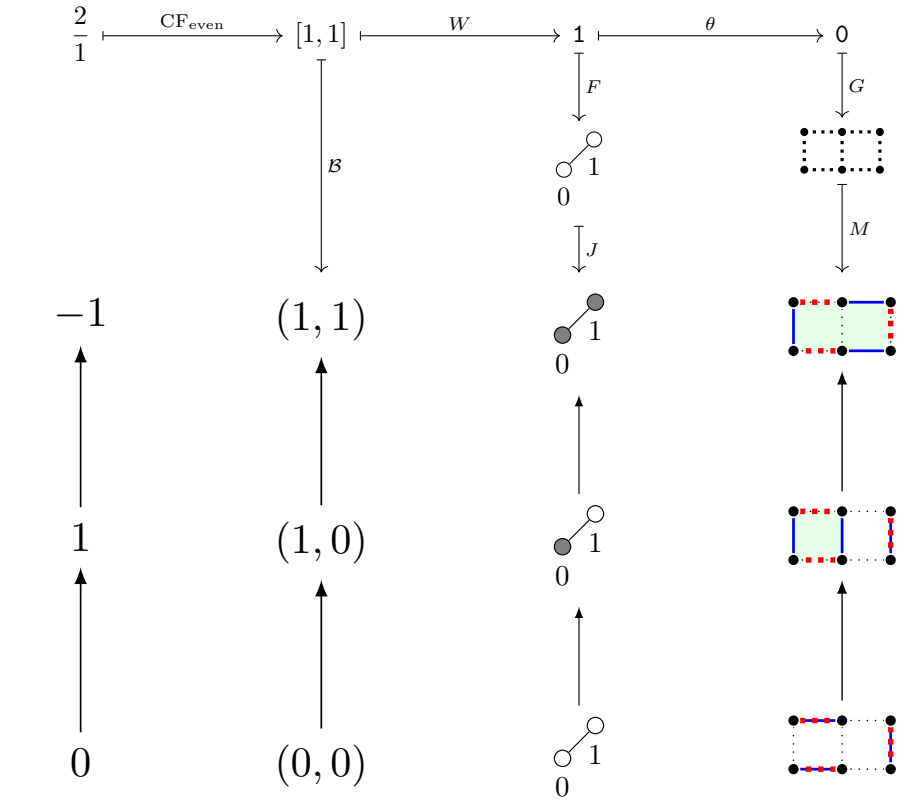
1


Z([0, 2])
= Z ∩ [−2, 1)
val
B([0, 2])
Ψ
J(1/2)
Φ
M(1/2)
(r0, −r1) = (1, −1)
r2 = 3
[1/2]q = q^−1 q^2/(q+1)


$\frac{2}{1}$
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$[1,1]$
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$(1,0)$
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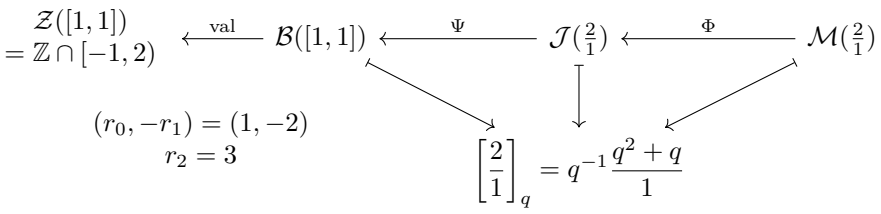
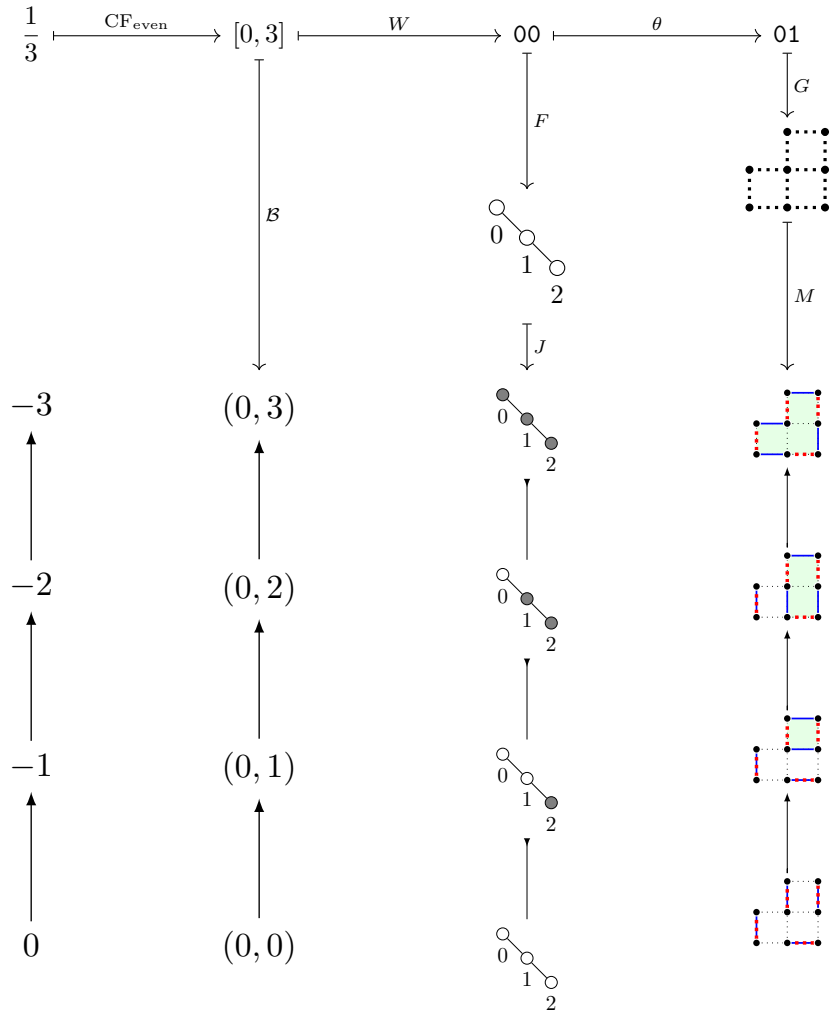
$(0,0)$


$\mathcal{Z}([1,1])$
$= \mathbb{Z} \cap [-1,2)$
val
$\mathcal{B}([1,1])$
Ψ
$\mathcal{J}(\frac{2}{1})$
Φ
$\mathcal{M}(\frac{2}{1})$
$(r_0, -r_1) = (1,-2)$
$r_2 = 3$
$\left[\frac{2}{1}\right]_q = q^{-1}\frac{q^2+q}{1}$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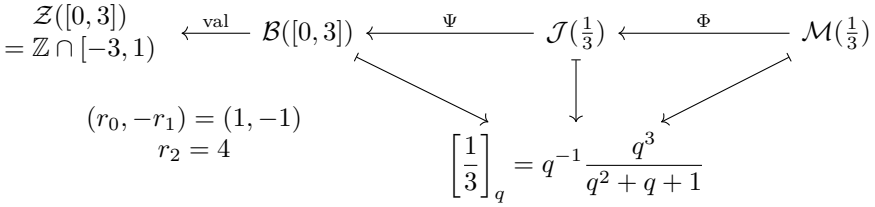
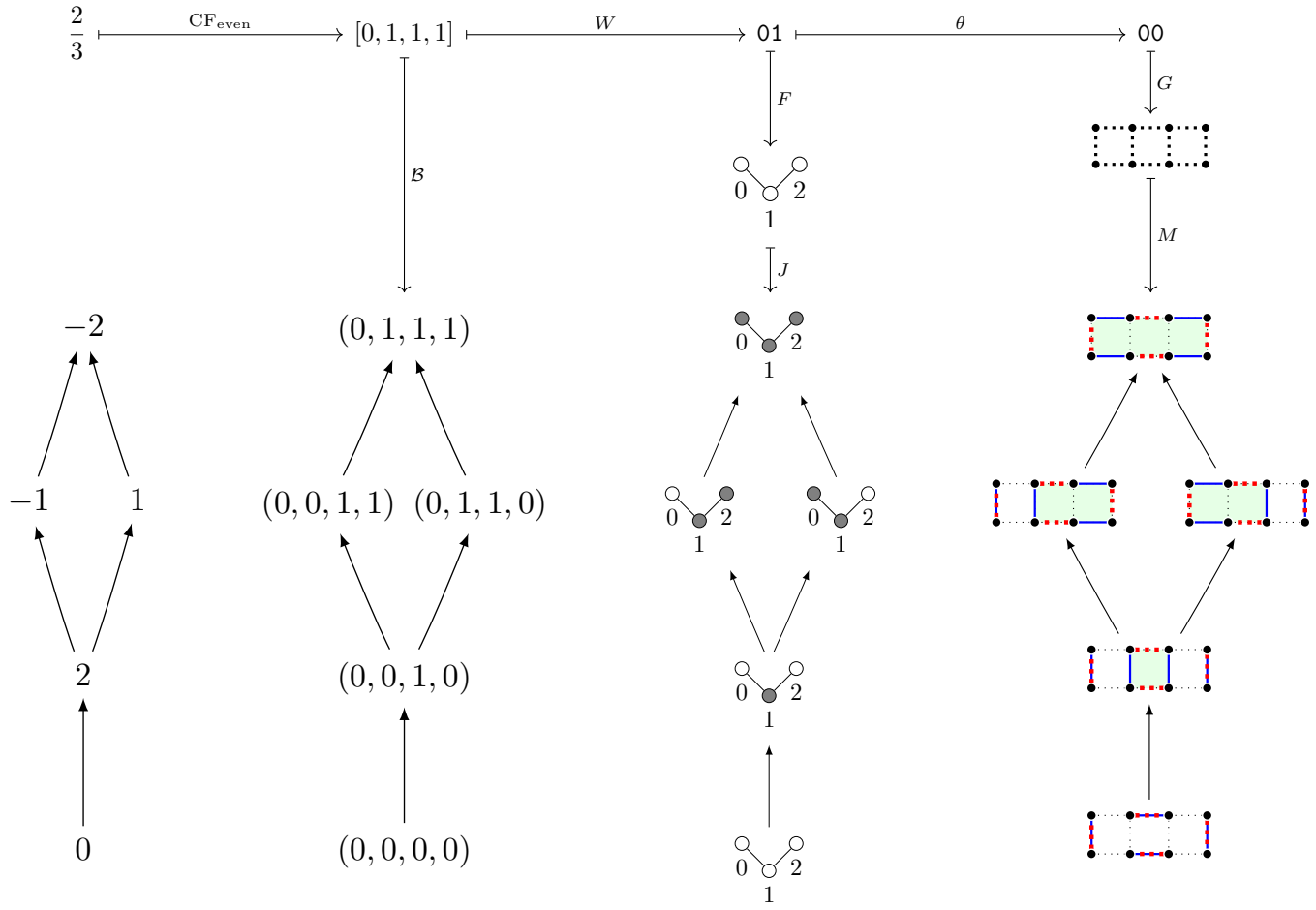
(0, 3)
(0, 2)
(0, 1)
(0, 0)


Z([0, 3])
= Z ∩ [−3, 1)
val
B([0, 3])
Ψ
J(1/3)
Φ
M(1/3)
(r0, −r1) = (1, −1)
r2 = 4
[1/3]q = q^−1 q^3/(q^2 + q + 1)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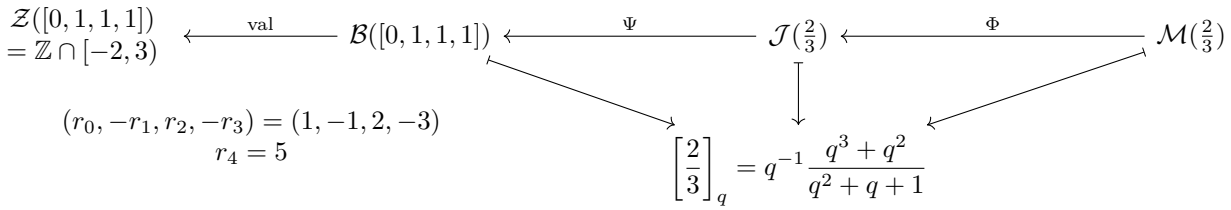
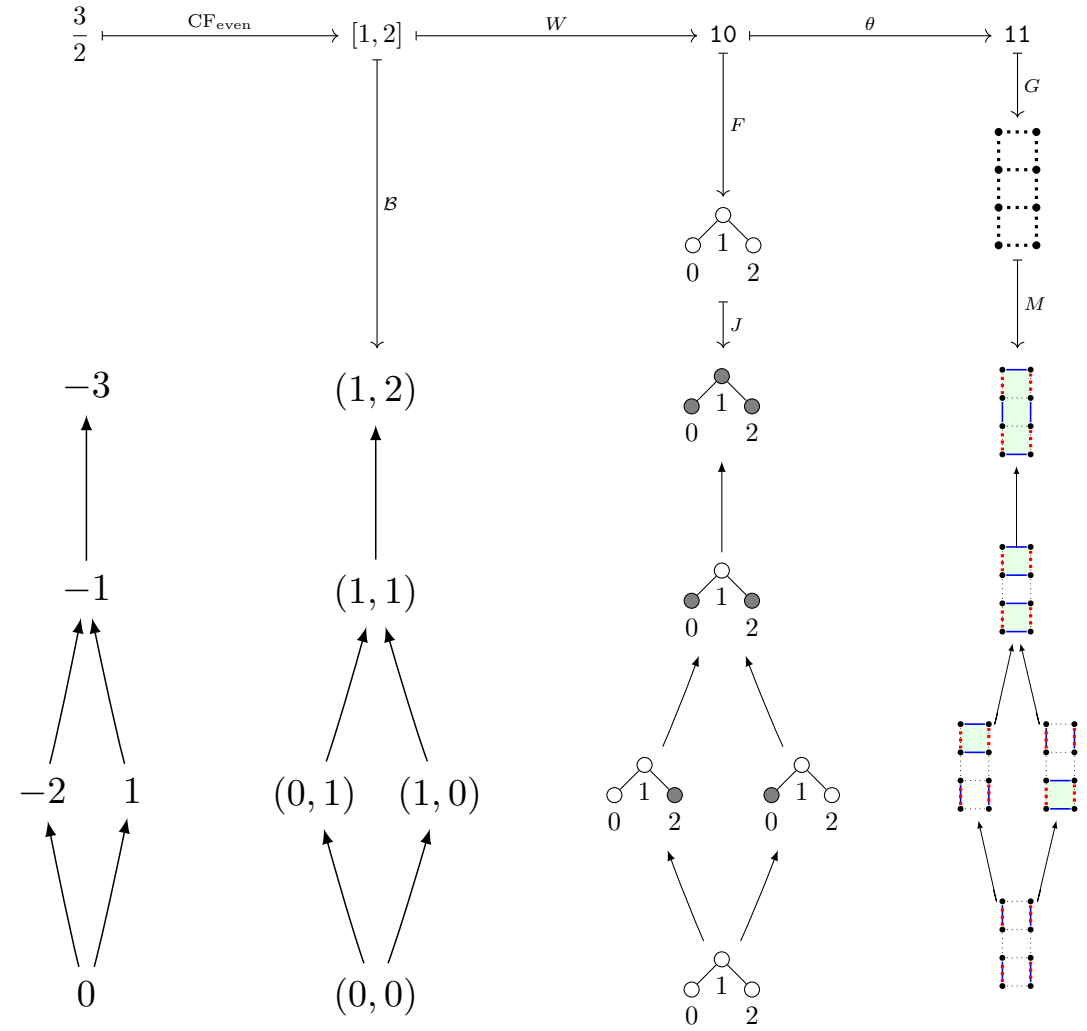
(0, 0, 1, 1)
(0, 1, 1, 0)
(0, 0, 1, 0)
(0, 0, 0, 0)

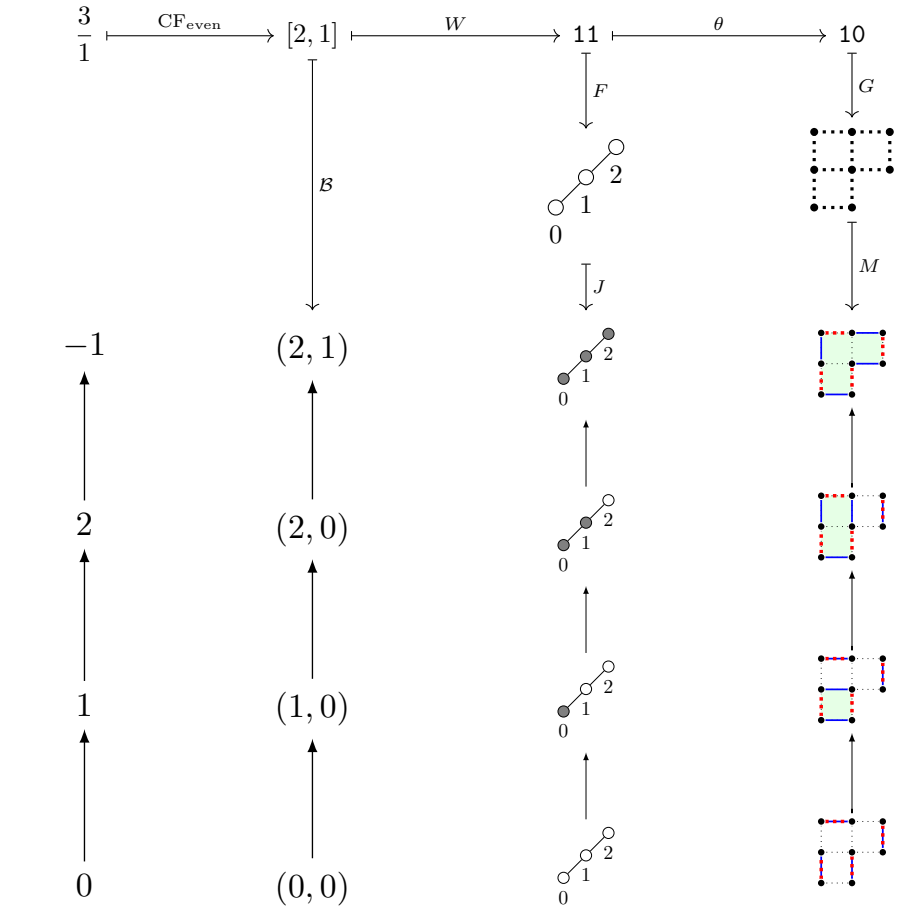

Z([0, 1, 1, 1])
= Z ∩ [−2, 3)
val
B([0, 1, 1, 1])
Ψ
J(2/3)
Φ
M(2/3)
(r0, −r1, r2, −r3) = (1, −1, 2, −3)
r4 = 5
[2/3]q = q^−1 (q^3 + q^2)/(q^2 + q + 1)



$$\mathcal{Z}([1,2]) = \mathbb{Z} \cap [-3,2) \xleftarrow{\text{val}} \mathcal{B}([1,2]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{3}{2}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{3}{2})$$

$$(r_0, -r_1) = (1, -2)$$
$$r_2 = 5$$

$$\left[\frac{3}{2}\right]_q = q^{-1}\frac{q^3+q^2+q}{q+1}$$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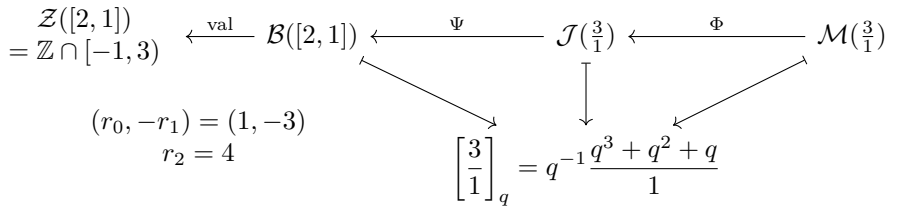
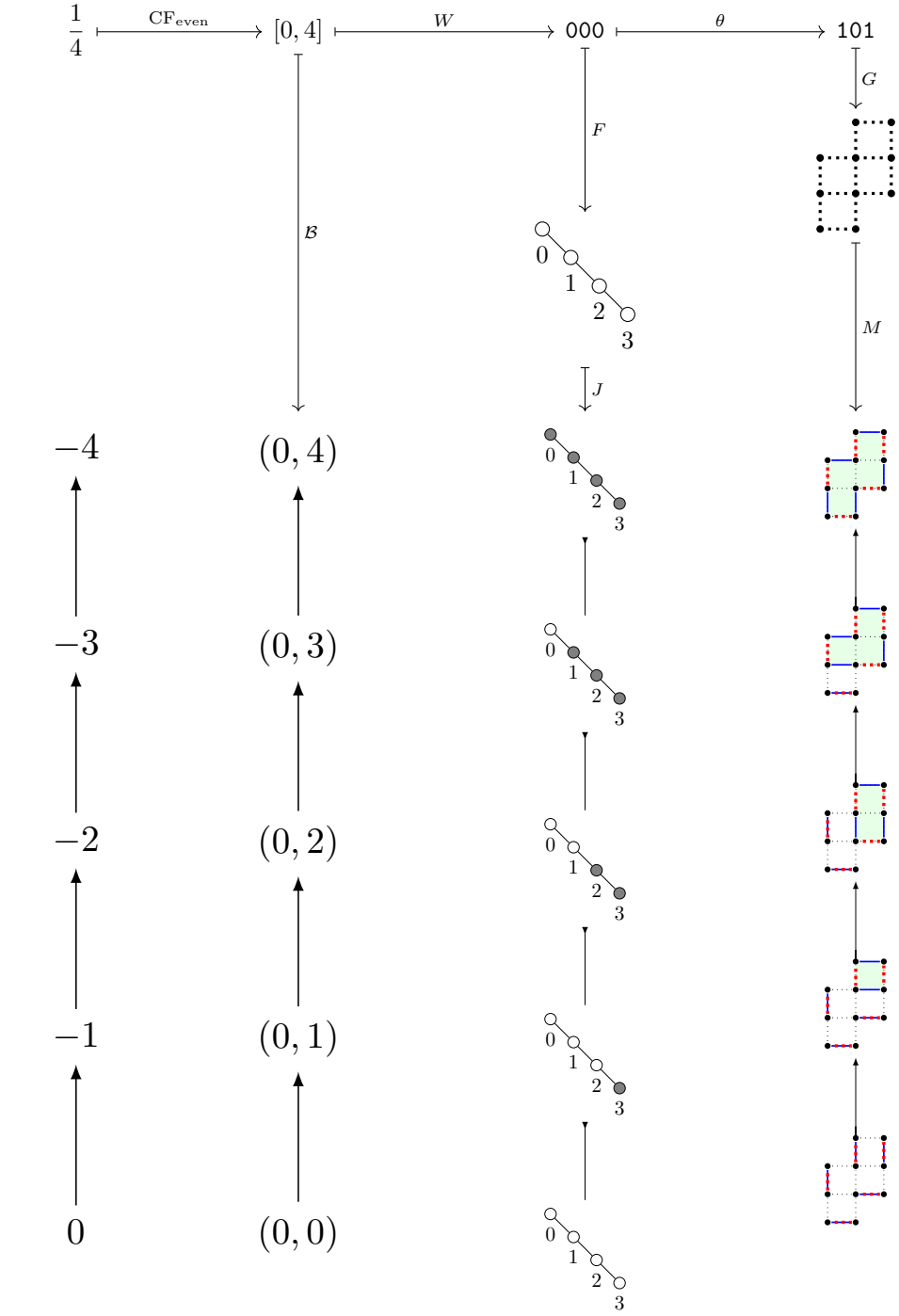
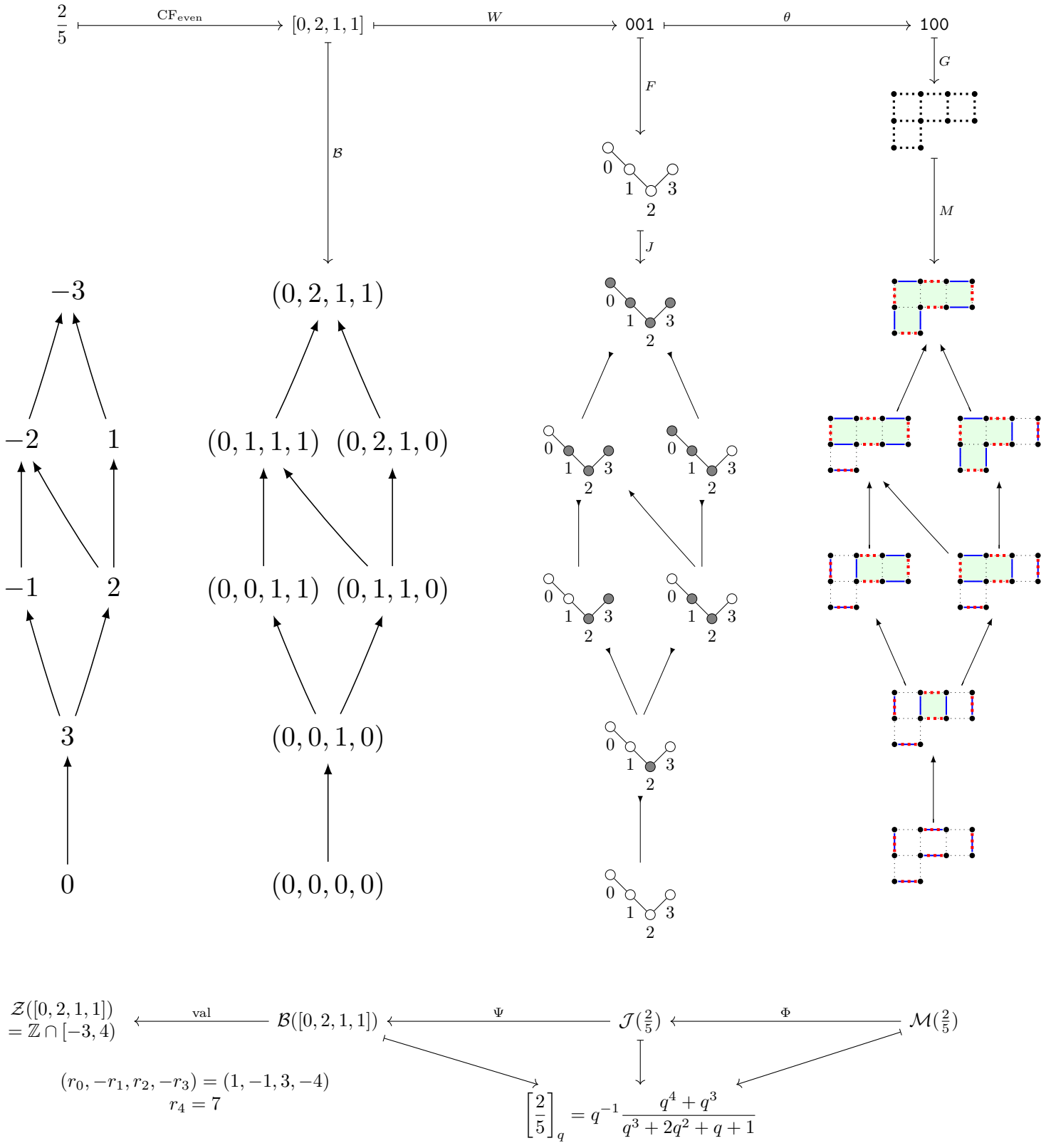

Z([2, 1])
= Z ∩ [−1, 3)
val
B([2, 1])
Ψ
J(3/1)
Φ
M(3/1)
(r0, −r1) = (1, −3)
r2 = 4
[3/1]q = q^−1 (q^3 + q^2 + q)/1



$$\begin{array}{ccccccc} \mathcal{Z}([0,4]) & \xleftarrow{\text{val}} & \mathcal{B}([0,4]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{1}{4}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{1}{4}) \\ = \mathbb{Z} \cap [-4, 1) & & & & & & \end{array}$$

$$(r_0, -r_1) = (1, -1)$$
$$r_2 = 5$$

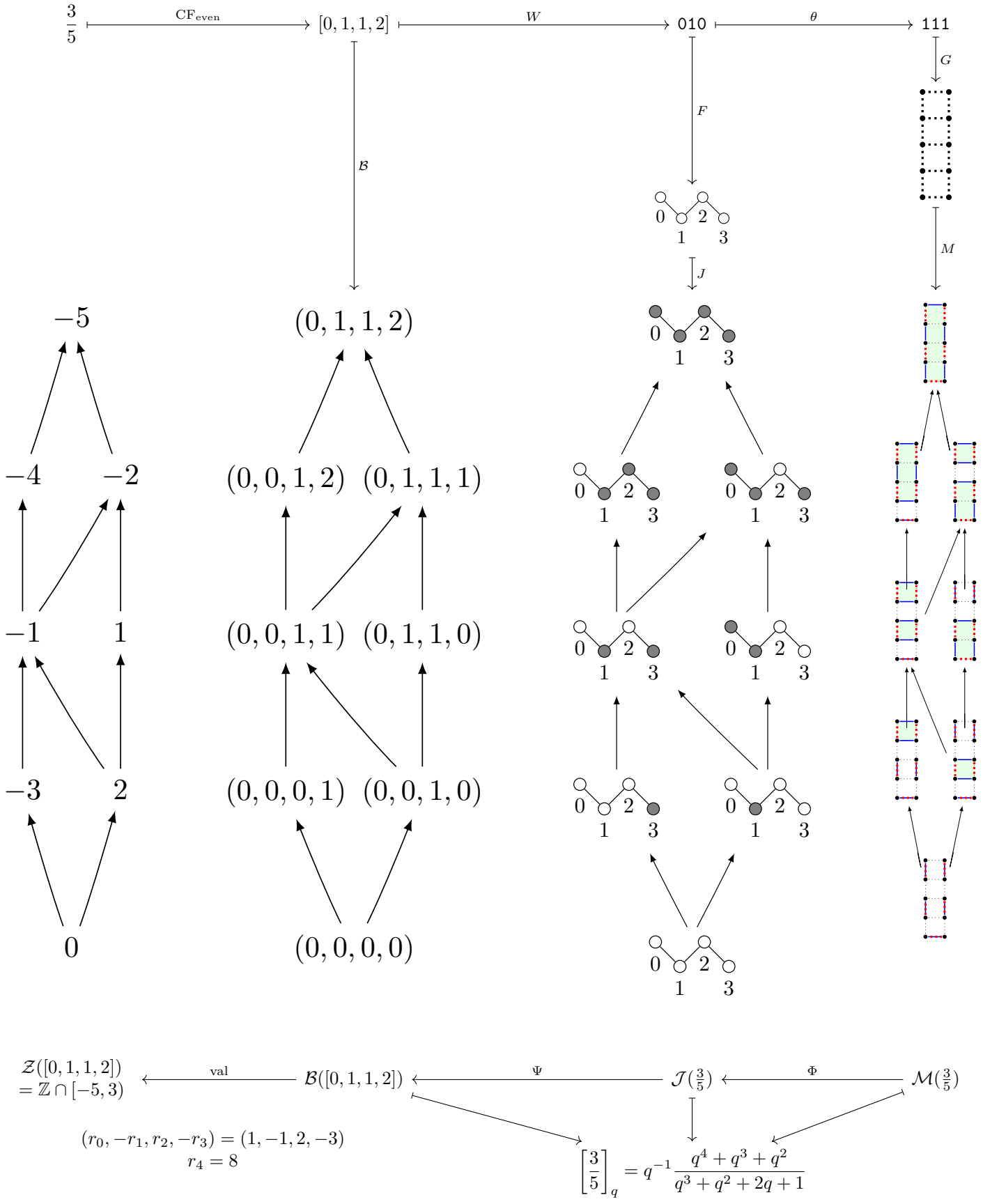
$$\left[\frac{1}{4}\right]_q = q^{-1}\frac{q^4}{q^3+q^2+q+1}$$



$\mathcal{Z}([0,2,1,1])$
$= \mathbb{Z} \cap [-3,4)$
$\xleftarrow{\mathrm{val}}$
$\mathcal{B}([0,2,1,1])$
$\xleftarrow{\Psi}$
$\mathcal{J}(\frac{2}{5})$
$\xleftarrow{\Phi}$
$\mathcal{M}(\frac{2}{5})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 3, -4)$
$r_4 = 7$

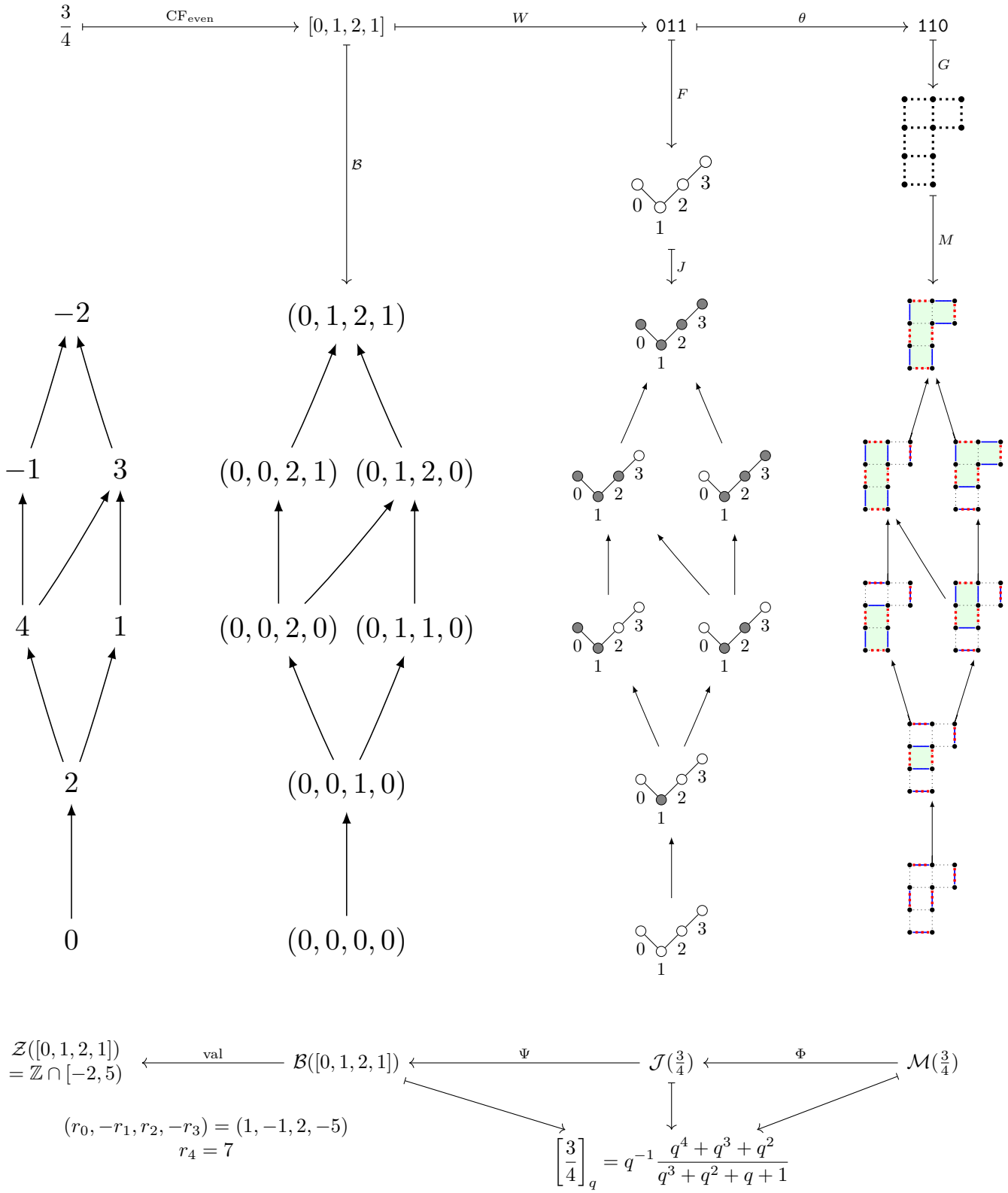
$$\left[\frac{2}{5}\right]_q = q^{-1}\frac{q^4+q^3}{q^3+2q^2+q+1}$$



$\mathcal{Z}([0,1,1,2])$
$= \mathbb{Z} \cap [-5,3)$
$\xleftarrow{\text{val}}$
$\mathcal{B}([0,1,1,2])$
$\xleftarrow{\Psi}$
$\mathcal{J}(\frac{3}{5})$
$\xleftarrow{\Phi}$
$\mathcal{M}(\frac{3}{5})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 2, -3)$
$r_4 = 8$

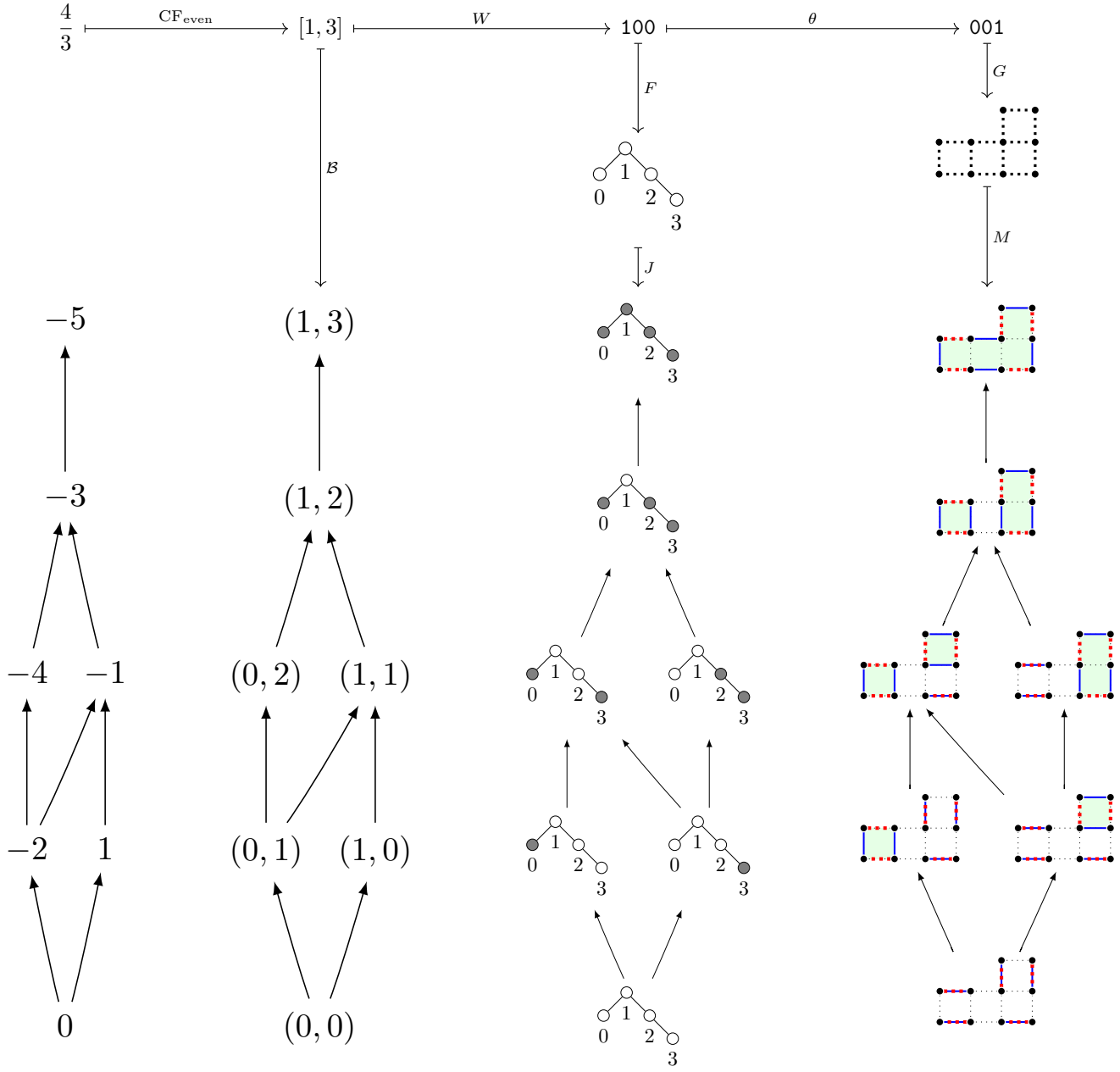
$$\left[\frac{3}{5}\right]_q = q^{-1}\frac{q^4+q^3+q^2}{q^3+q^2+2q+1}$$



$\mathcal{Z}([0,1,2,1])$
$= \mathbb{Z} \cap [-2,5)$
val
$\mathcal{B}([0,1,2,1])$
Ψ
$\mathcal{J}(\frac{3}{4})$
Φ
$\mathcal{M}(\frac{3}{4})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 2, -5)$
$r_4 = 7$

$$\left[\frac{3}{4}\right]_q = q^{-1}\frac{q^4+q^3+q^2}{q^3+q^2+q+1}$$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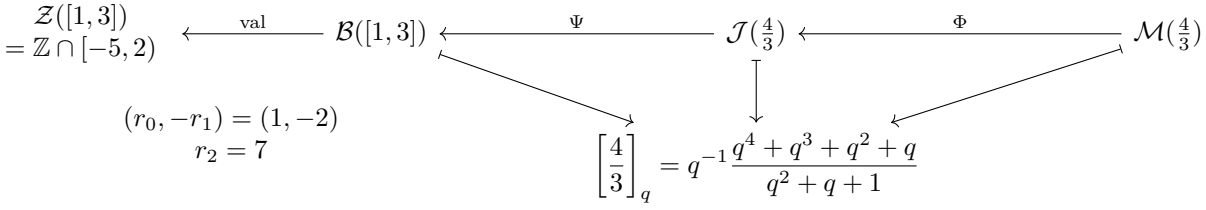
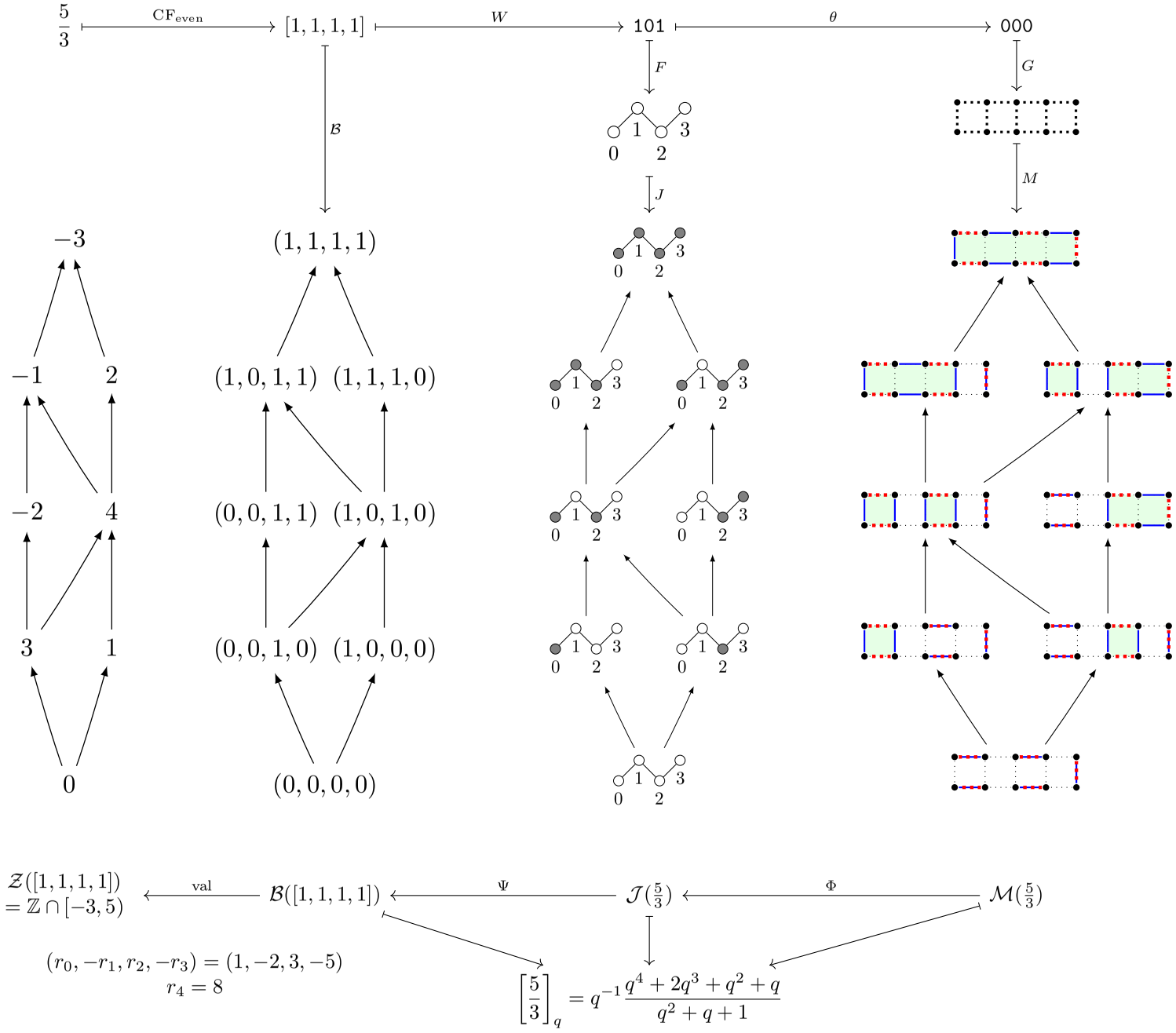
(0, 1)
(1, 0)
0
(0, 0)
0 1 2 3


Z([1, 3])
= Z ∩ [−5, 2)
val
B([1, 3])
Ψ
J(4/3)
Φ
M(4/3)
(r0, −r1) = (1, −2)
r2 = 7
[4/3]q = q^−1 (q^4 + q^3 + q^2 + q)/(q^2 + q + 1)


$\frac{5}{3}$
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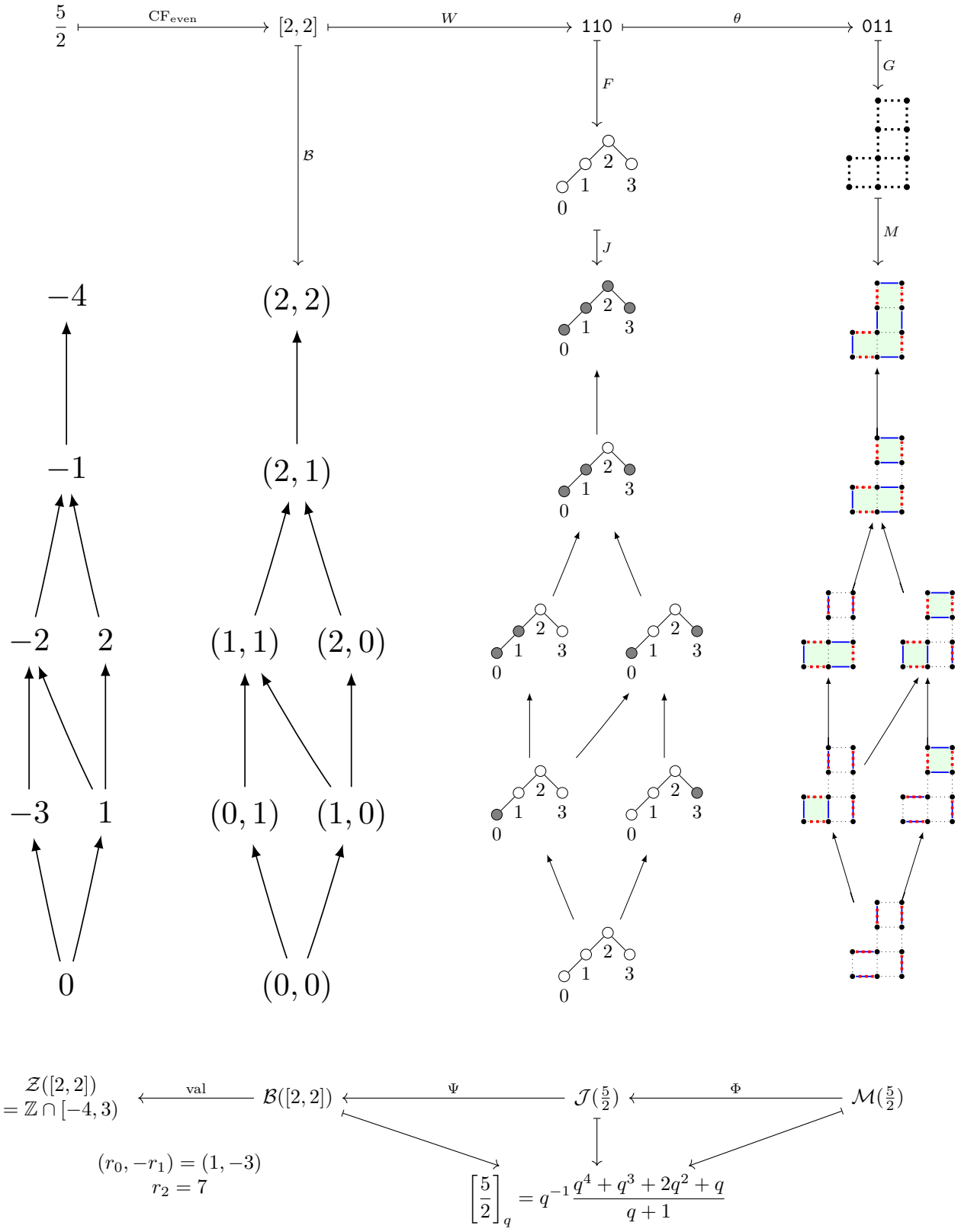
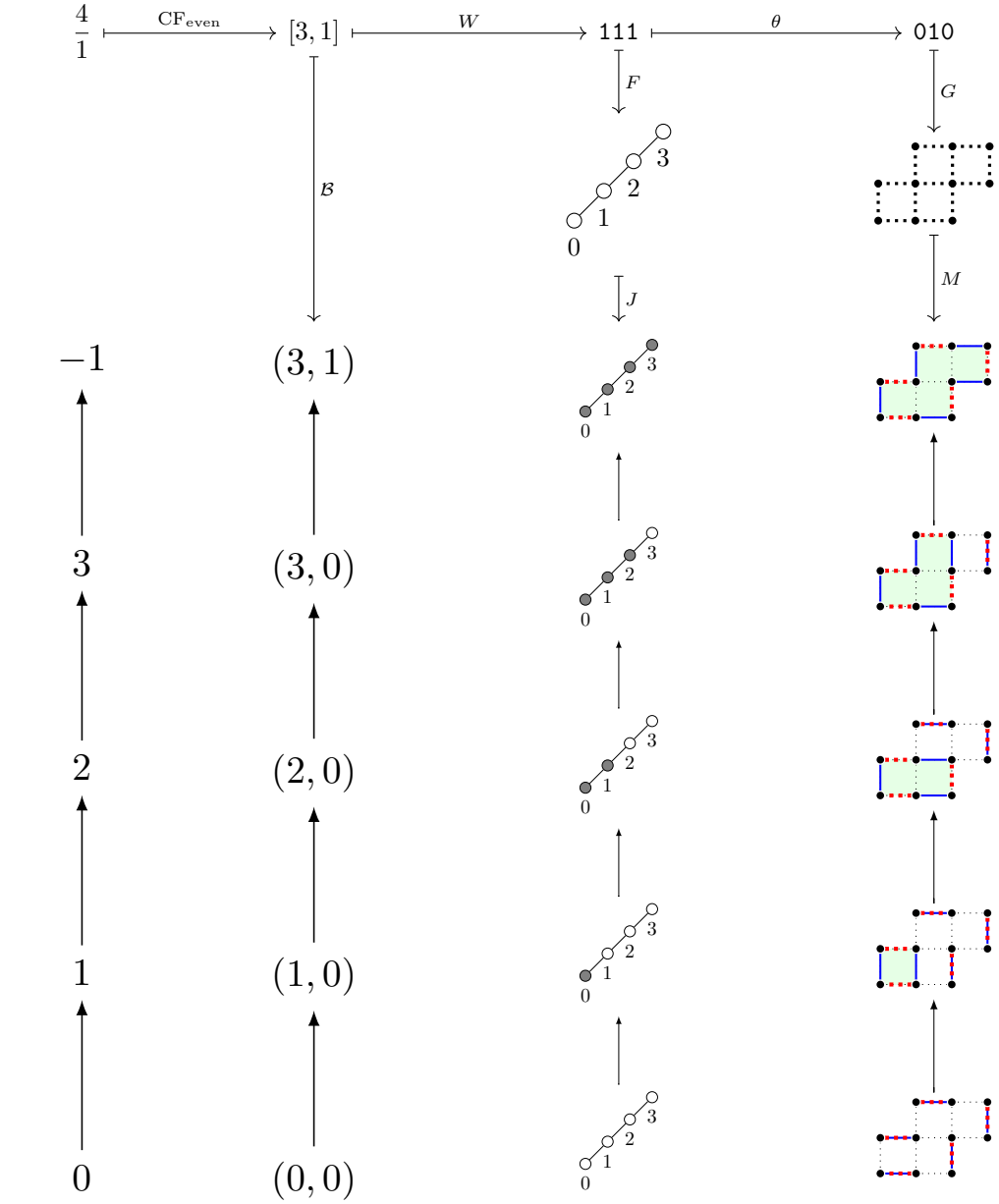
$(1,1,1,1)$
$(1,0,1,1)$
$(1,1,1,0)$
$(0,0,1,1)$
$(1,0,1,0)$
$(0,0,1,0)$
$(1,0,0,0)$
$(0,0,0,0)$
$\mathcal{Z}([1,1,1,1])$
$= \mathbb{Z} \cap [-3,5)$
val
$\mathcal{B}([1,1,1,1])$
Ψ
$\mathcal{J}(\frac{5}{3})$
Φ
$\mathcal{M}(\frac{5}{3})$
$(r_0, -r_1, r_2, -r_3) = (1, -2, 3, -5)$
$r_4 = 8$
$\left[\frac{5}{3}\right]_q = q^{-1}\frac{q^4+2q^3+q^2+q}{q^2+q+1}$



$$\mathcal{Z}([2,2]) = \mathbb{Z} \cap [-4,3) \xleftarrow{\text{val}} \mathcal{B}([2,2]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{5}{2}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{5}{2})$$

$$(r_0, -r_1) = (1,-3)$$
$$r_2 = 7$$

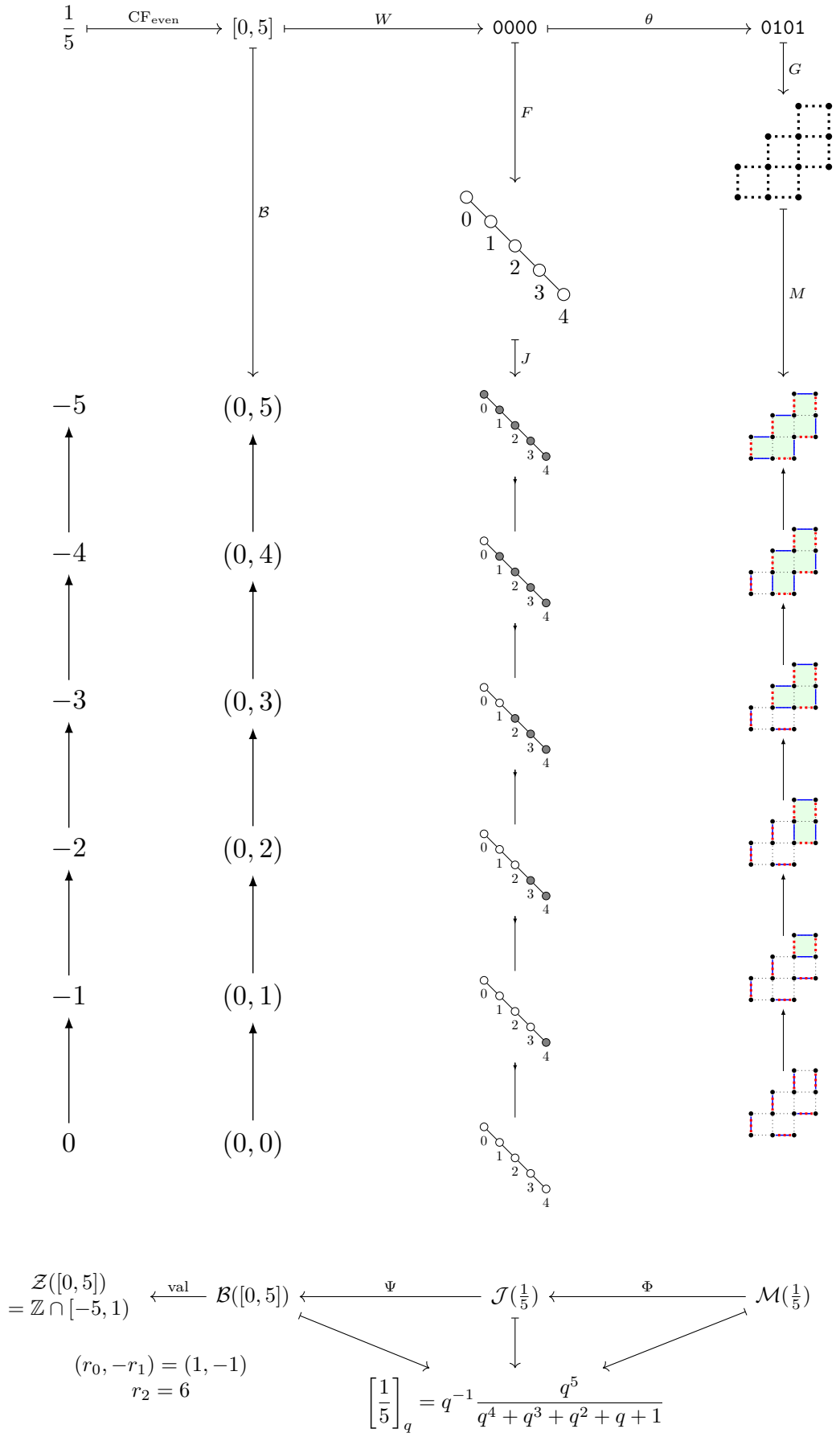
$$\left[\frac{5}{2}\right]_q = q^{-1}\frac{q^4+q^3+2q^2+q}{q+1}$$



$$\underset{=\mathbb{Z}\cap[-1,4)}{\mathcal{Z}([3,1])} \xleftarrow{\text{val}} \mathcal{B}([3,1]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{4}{1}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{4}{1})$$

$$(r_0, -r_1) = (1, -4)$$
$$r_2 = 5$$

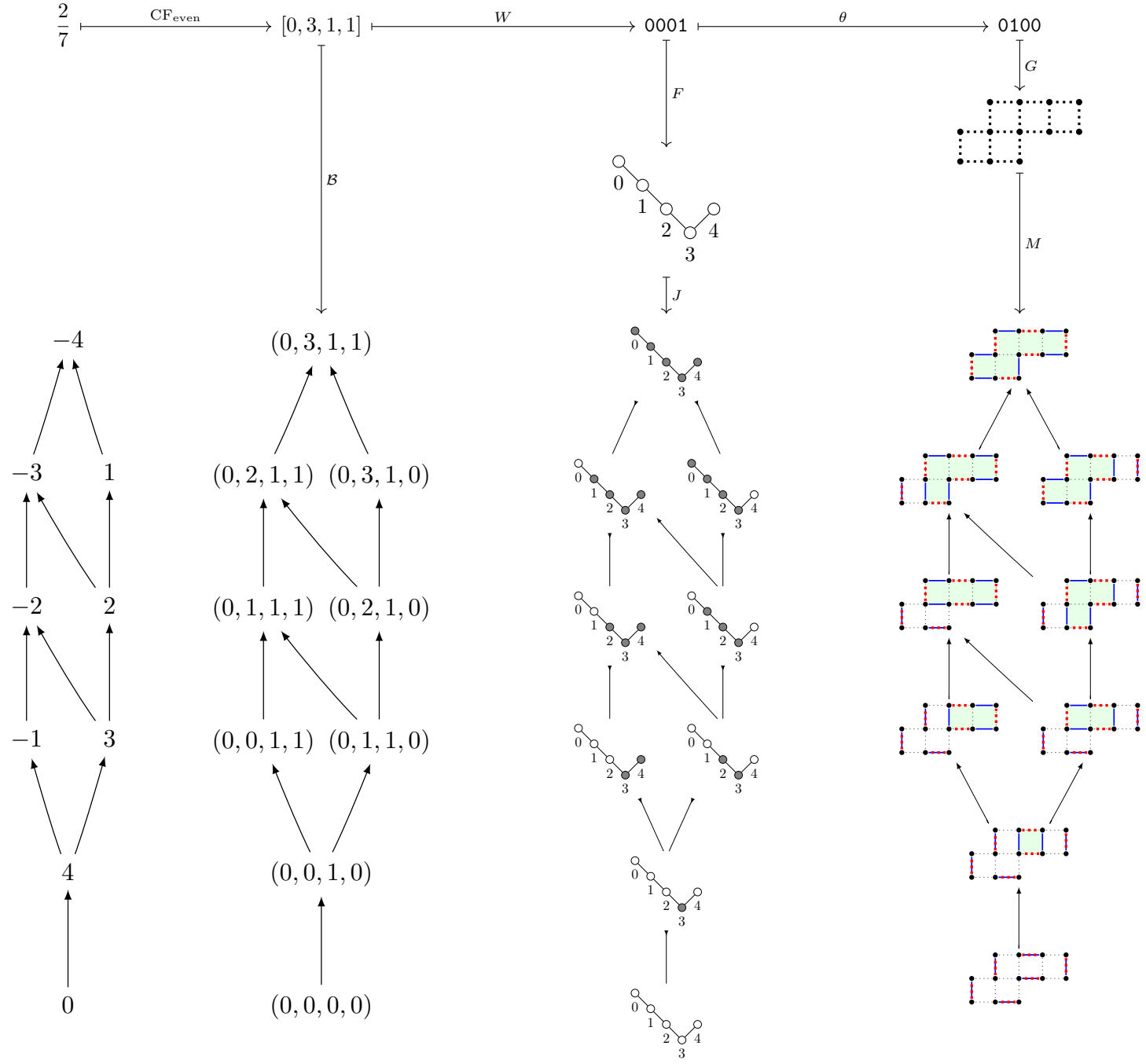
$$\left[\frac{4}{1}\right]_q = q^{-1}\frac{q^4+q^3+q^2+q}{1}$$



$$\underset{=\mathbb{Z}\cap[-5,1)}{\mathcal{Z}([0,5])} \xleftarrow{\text{val}} \mathcal{B}([0,5]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{1}{5}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{1}{5})$$

$$(r_0, -r_1) = (1, -1)$$
$$r_2 = 6$$

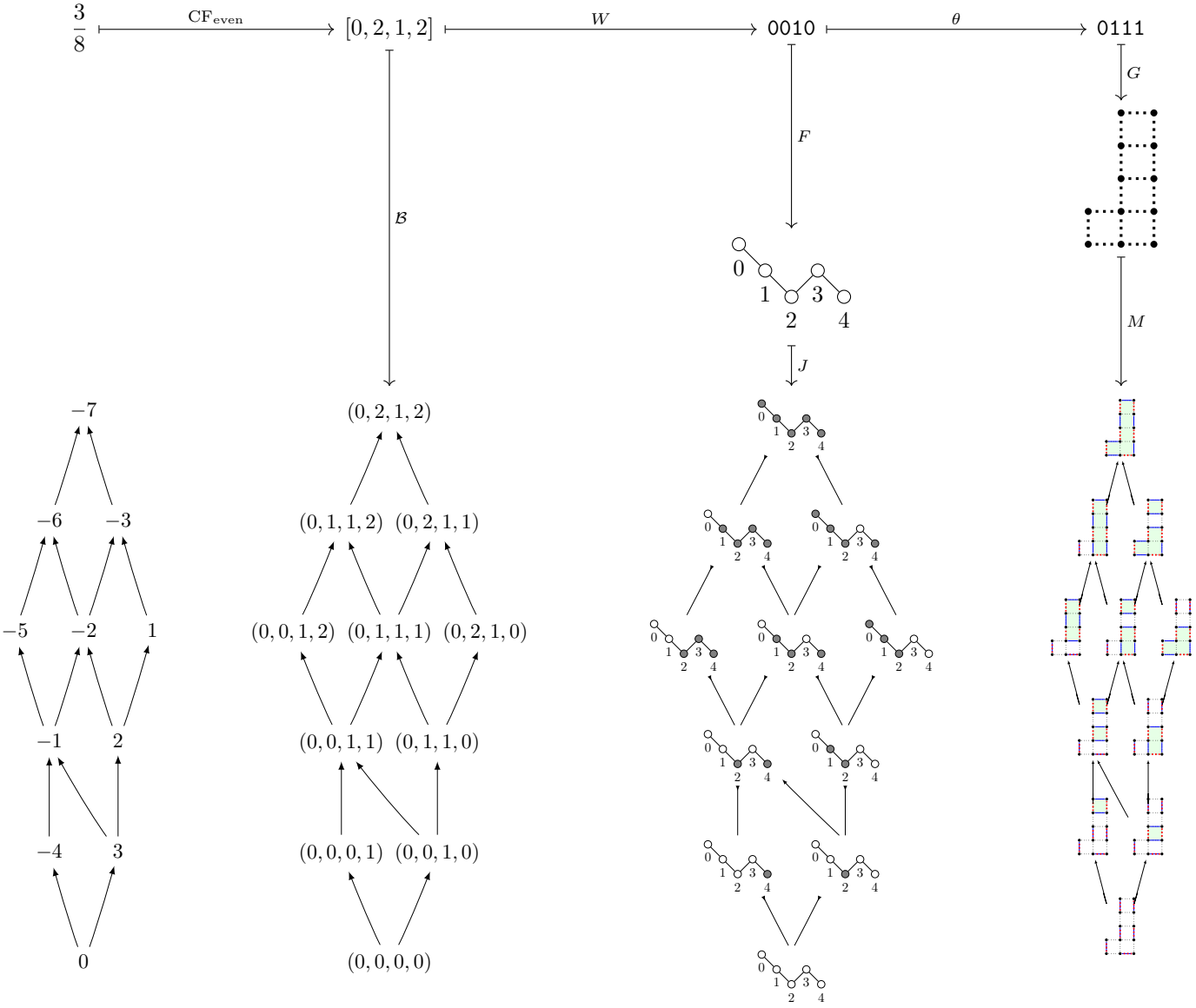
$$\left[\frac{1}{5}\right]_q = q^{-1}\frac{q^5}{q^4+q^3+q^2+q+1}$$



$\mathcal{Z}([0,3,1,1]) = \mathbb{Z} \cap [-4,5)$ $\xleftarrow{\text{val}}$ $\mathcal{B}([0,3,1,1])$ $\xleftarrow{\Psi}$ $\mathcal{J}(\frac{2}{7})$ $\xleftarrow{\Phi}$ $\mathcal{M}(\frac{2}{7})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 4, -5)$
$r_4 = 9$

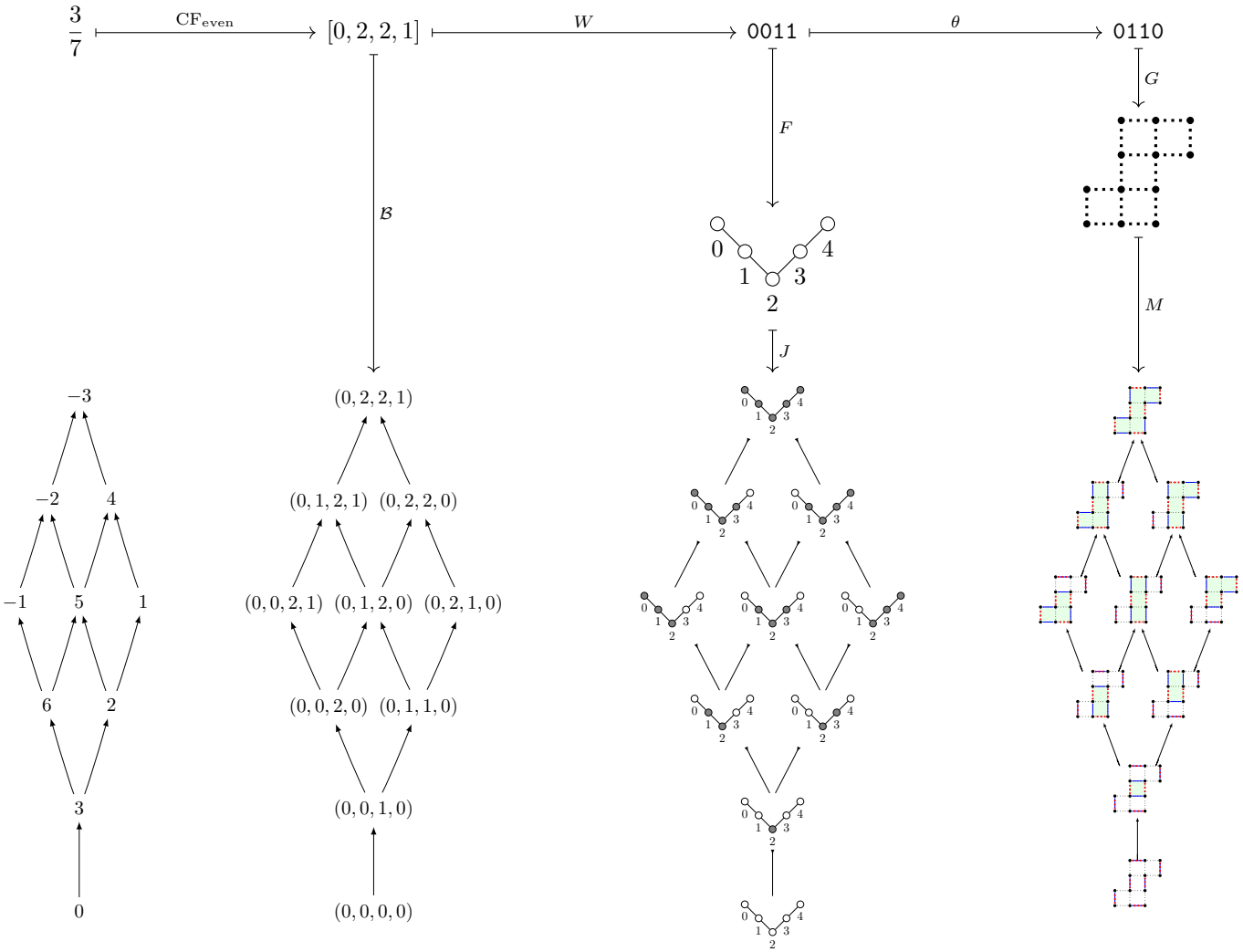
$$\left[\frac{2}{7}\right]_q = q^{-1}\frac{q^5+q^4}{q^4+2q^3+2q^2+q+1}$$



$\mathcal{Z}([0,2,1,2])$
$= \mathbb{Z} \cap [-7, 4)$
$\xleftarrow{\text{val}}$
$\mathcal{B}([0,2,1,2])$
$\xleftarrow{\Psi}$
$\mathcal{J}(\frac{3}{8})$
$\xleftarrow{\Phi}$
$\mathcal{M}(\frac{3}{8})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 3, -4)$
$r_4 = 11$

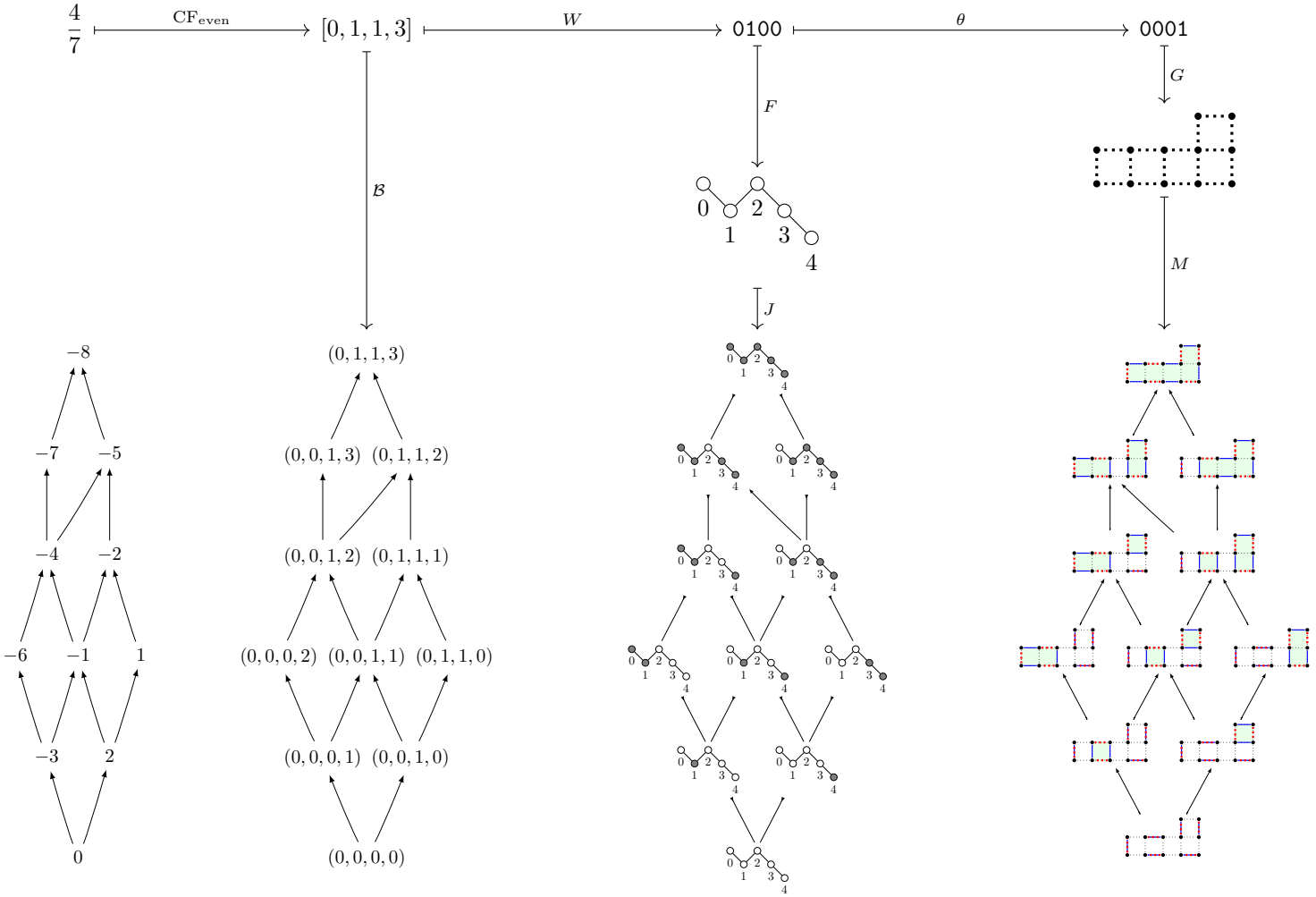
$$\left[\frac{3}{8}\right]_q = q^{-1} \frac{q^5 + q^4 + q^3}{q^4 + 2q^3 + 2q^2 + 2q + 1}$$



$\mathcal{Z}([0,2,2,1])$
$= \mathbb{Z} \cap [-3, 7)$
$\xleftarrow{\text{val}}$ $\mathcal{B}([0,2,2,1])$ $\xleftarrow{\Psi}$ $\mathcal{J}(\frac{3}{7})$ $\xleftarrow{\Phi}$ $\mathcal{M}(\frac{3}{7})$

$(r_0, -r_1, r_2, -r_3) = (1, -1, 3, -7)$
$r_4 = 10$

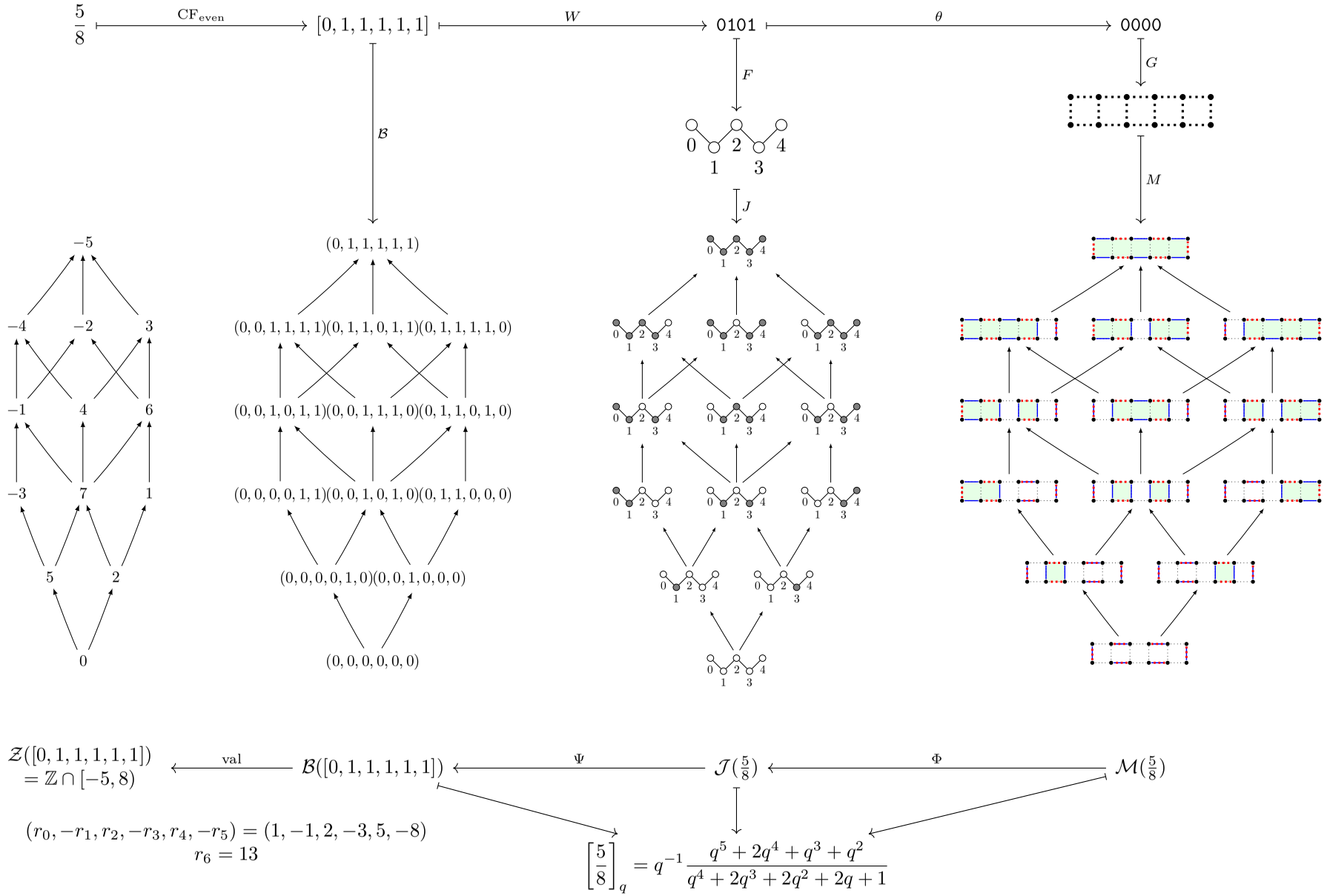
$$\left[\frac{3}{7}\right]_q = q^{-1} \frac{q^5 + q^4 + q^3}{q^4 + 2q^3 + 2q^2 + q + 1}$$



$$\mathcal{Z}([0,1,1,3]) = \mathbb{Z} \cap [-8,3) \xleftarrow{\text{val}} \mathcal{B}([0,1,1,3]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{4}{7}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{4}{7})$$

$$(r_0, -r_1, r_2, -r_3) = (1,-1,2,-3)$$
$$r_4 = 11$$

$$\left[\frac{4}{7}\right]_q = q^{-1}\frac{q^5+q^4+q^3+q^2}{q^4+q^3+2q^2+2q+1}$$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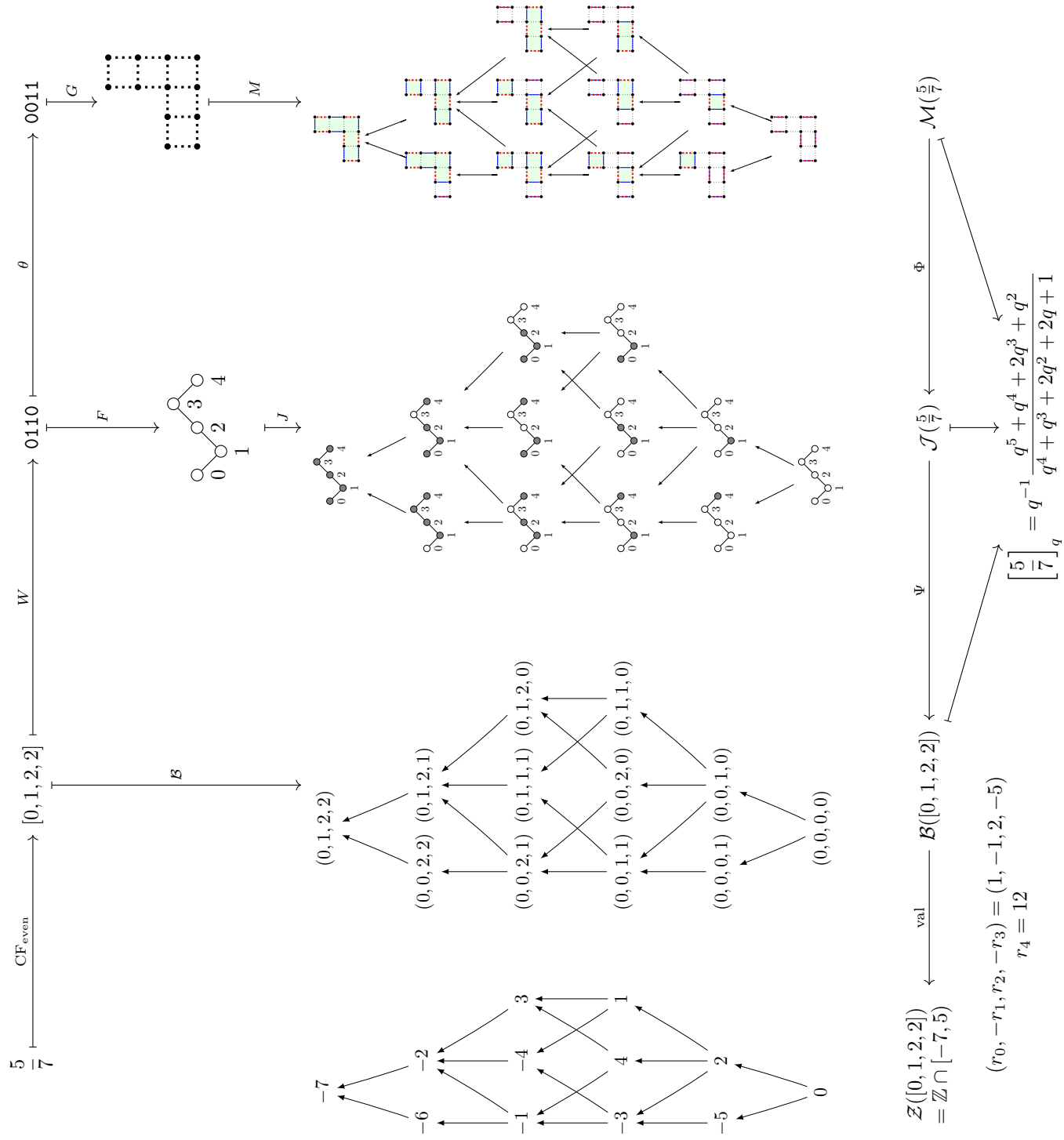
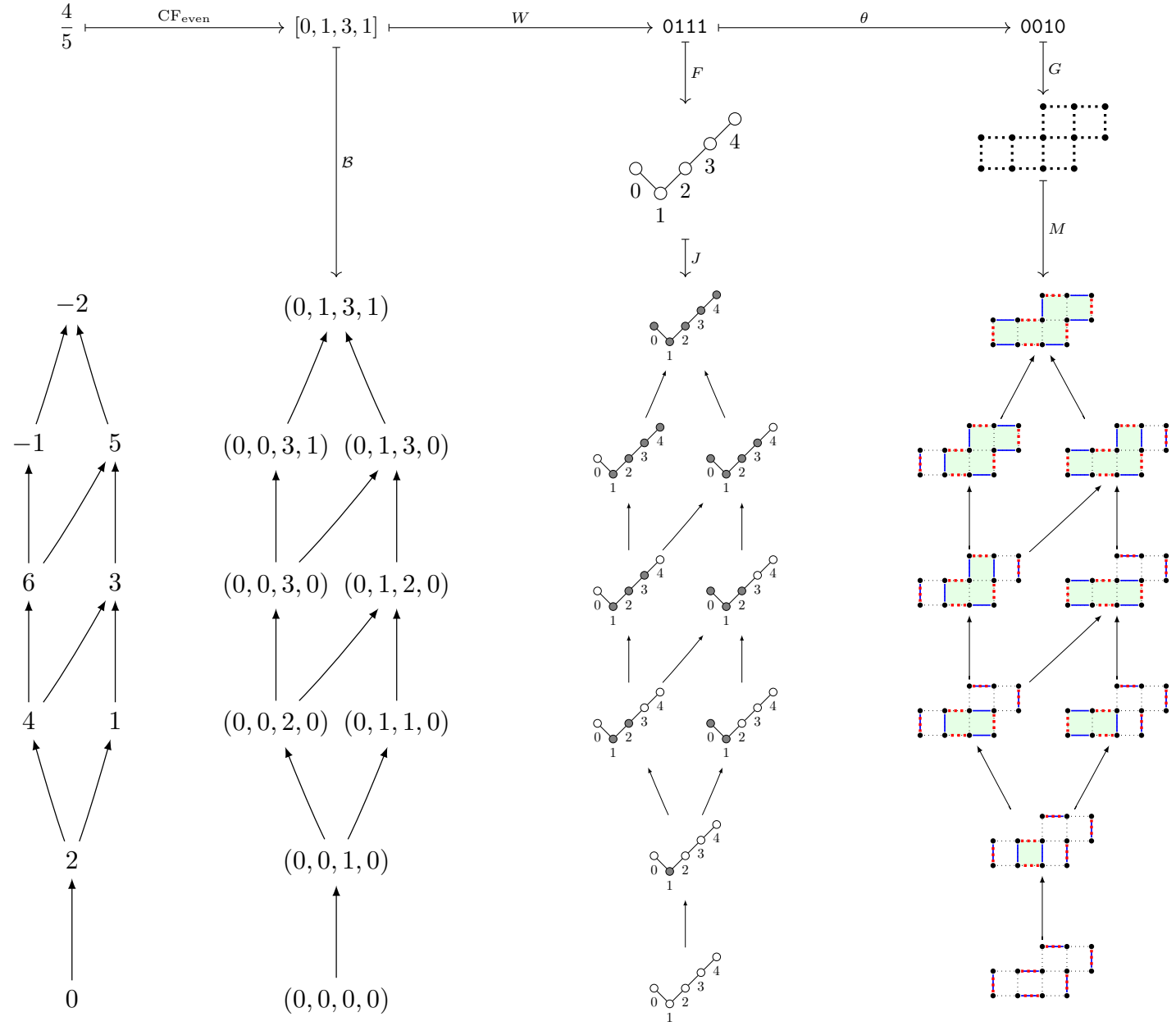
$(0,0,1,1,1,1)(0,1,1,0,1,1)(0,1,1,1,1,0)$
$(0,0,1,0,1,1)(0,0,1,1,1,0)(0,1,1,0,1,0)$
$(0,0,0,0,1,1)(0,0,1,0,1,0)(0,1,1,0,0,0)$
$(0,0,0,0,1,0)(0,0,1,0,0,0)$
$(0,0,0,0,0,0)$
$\mathcal{Z}([0,1,1,1,1,1])$
$= \mathbb{Z} \cap [-5,8)$
val
$\mathcal{B}([0,1,1,1,1,1])$
Ψ
$\mathcal{J}(\frac{5}{8})$
Φ
$\mathcal{M}(\frac{5}{8})$
$(r_0, -r_1, r_2, -r_3, r_4, -r_5) = (1, -1, 2, -3, 5, -8)$
$r_6 = 13$
$\left[\frac{5}{8}\right]_q = q^{-1} \frac{q^5 + 2q^4 + q^3 + q^2}{q^4 + 2q^3 + 2q^2 + 2q + 1}$

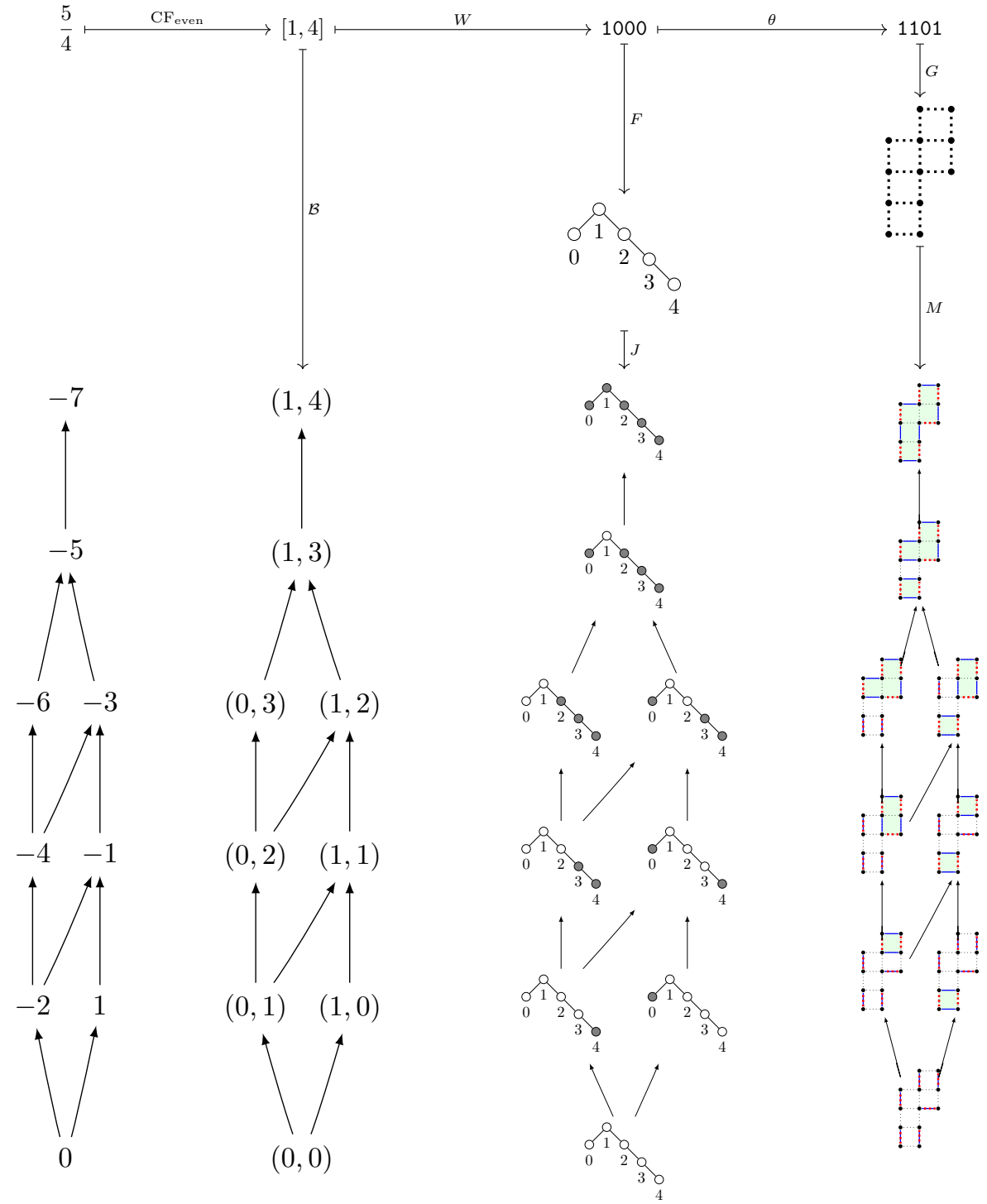

$\frac{5}{7}$
$\mathrm{CF}_{\mathrm{even}}$
$[0,1,2,2]$
$\mathcal{B}$
W
0110
F
J
θ
0011
G
M
$\mathcal{Z}([0,1,2,2])$
$= \mathbb{Z} \cap [-7,5)$
val
$\mathcal{B}([0,1,2,2])$
Ψ
$\mathcal{J}(\frac{5}{7})$
Φ
$\mathcal{M}(\frac{5}{7})$
$(r_0, -r_1, r_2, -r_3) = (1, -1, 2, -5)$
$r_4 = 12$
$\left[\frac{5}{7}\right]_q = q^{-1} \frac{q^5 + q^4 + 2q^3 + q^2}{q^4 + q^3 + 2q^2 + 2q + 1}$

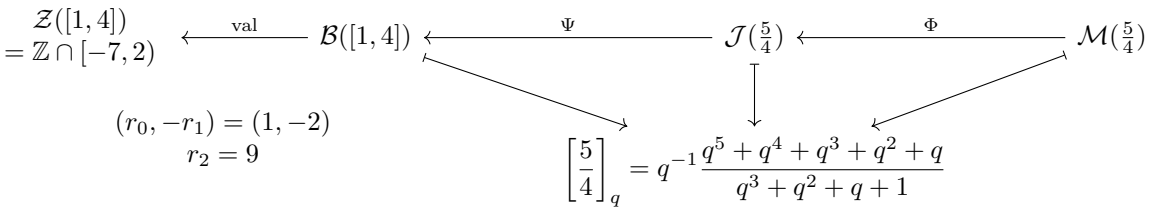


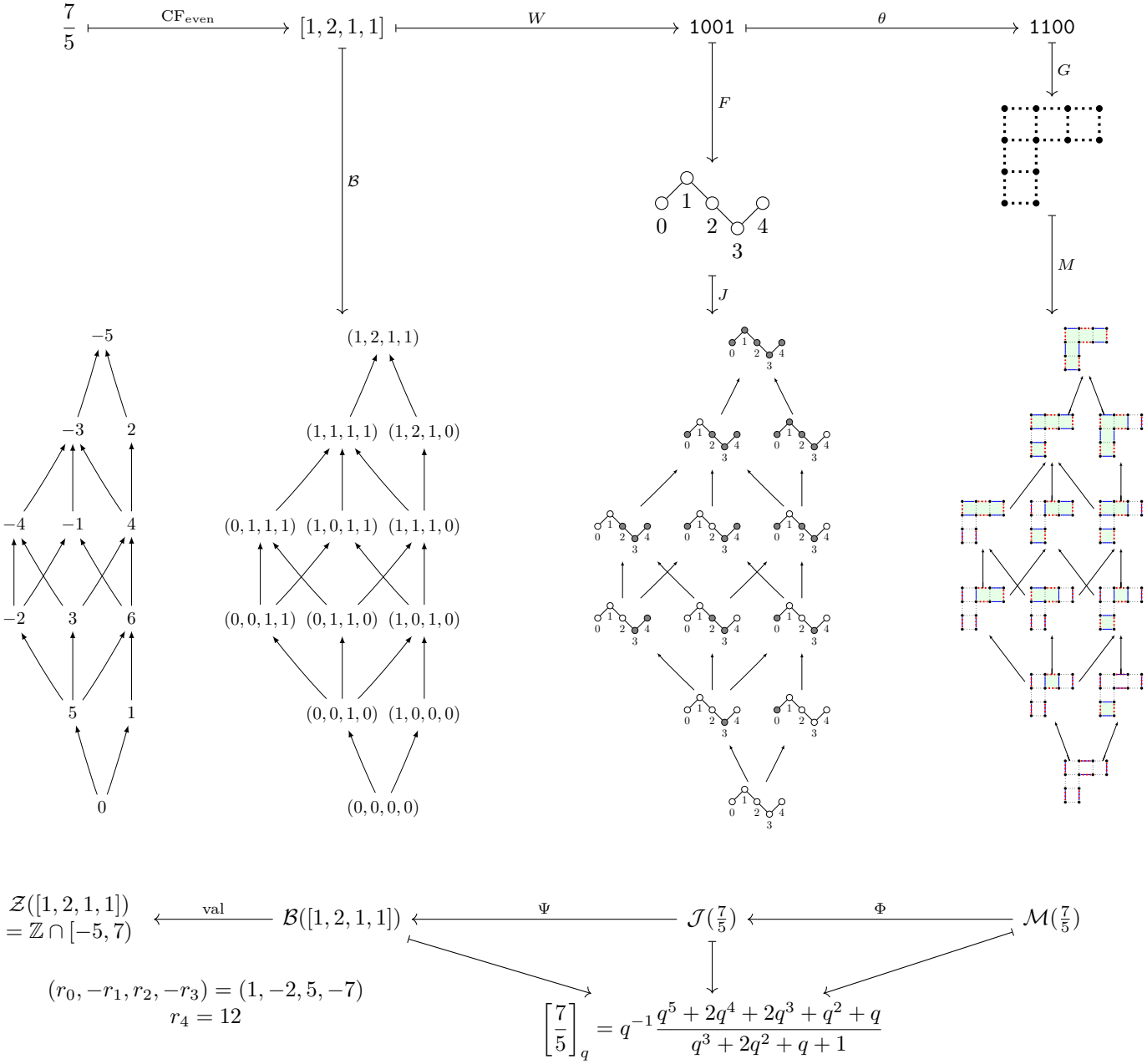
$$\mathcal{Z}([0,1,3,1]) = \mathbb{Z} \cap [-2,7) \xleftarrow{\text{val}} \mathcal{B}([0,1,3,1]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{4}{5}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{4}{5})$$

$$(r_0, -r_1, r_2, -r_3) = (1, -1, 2, -7)$$
$$r_4 = 9$$

$$\left[\frac{4}{5}\right]_q = q^{-1}\frac{q^5+q^4+q^3+q^2}{q^4+q^3+q^2+q+1}$$


5/4
CF_even
[1, 4]
W
1000
θ
1101
G
F
B
M
J
−7
−5
−6
−3
−4
−1
−2
1
0
(1, 4)
(1, 3)
(0, 3)
(1, 2)
(0, 2)
(1, 1)
(0, 1)
(1, 0)
(0, 0)

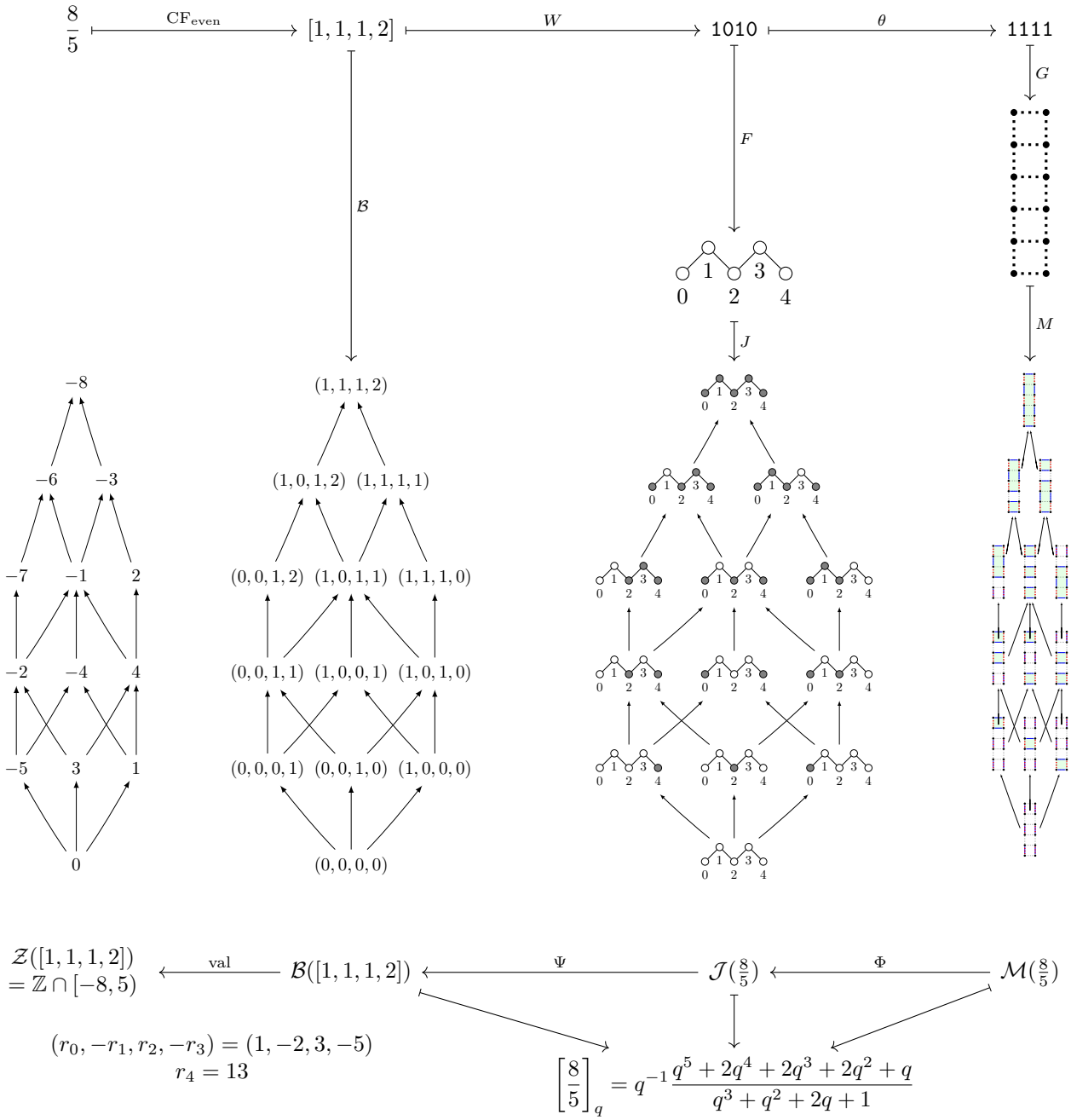

Z([1, 4])
= Z ∩ [−7, 2)
val
B([1, 4])
Ψ
J(5/4)
Φ
M(5/4)
(r0, −r1) = (1, −2)
r2 = 9
[5/4]q = q^−1 (q^5 + q^4 + q^3 + q^2 + q) / (q^3 + q^2 + q + 1)



$$\mathcal{Z}([1,2,1,1]) = \mathbb{Z} \cap [-5,7) \xleftarrow{\text{val}} \mathcal{B}([1,2,1,1]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{7}{5}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{7}{5})$$

$$(r_0, -r_1, r_2, -r_3) = (1, -2, 5, -7)$$
$$r_4 = 12$$

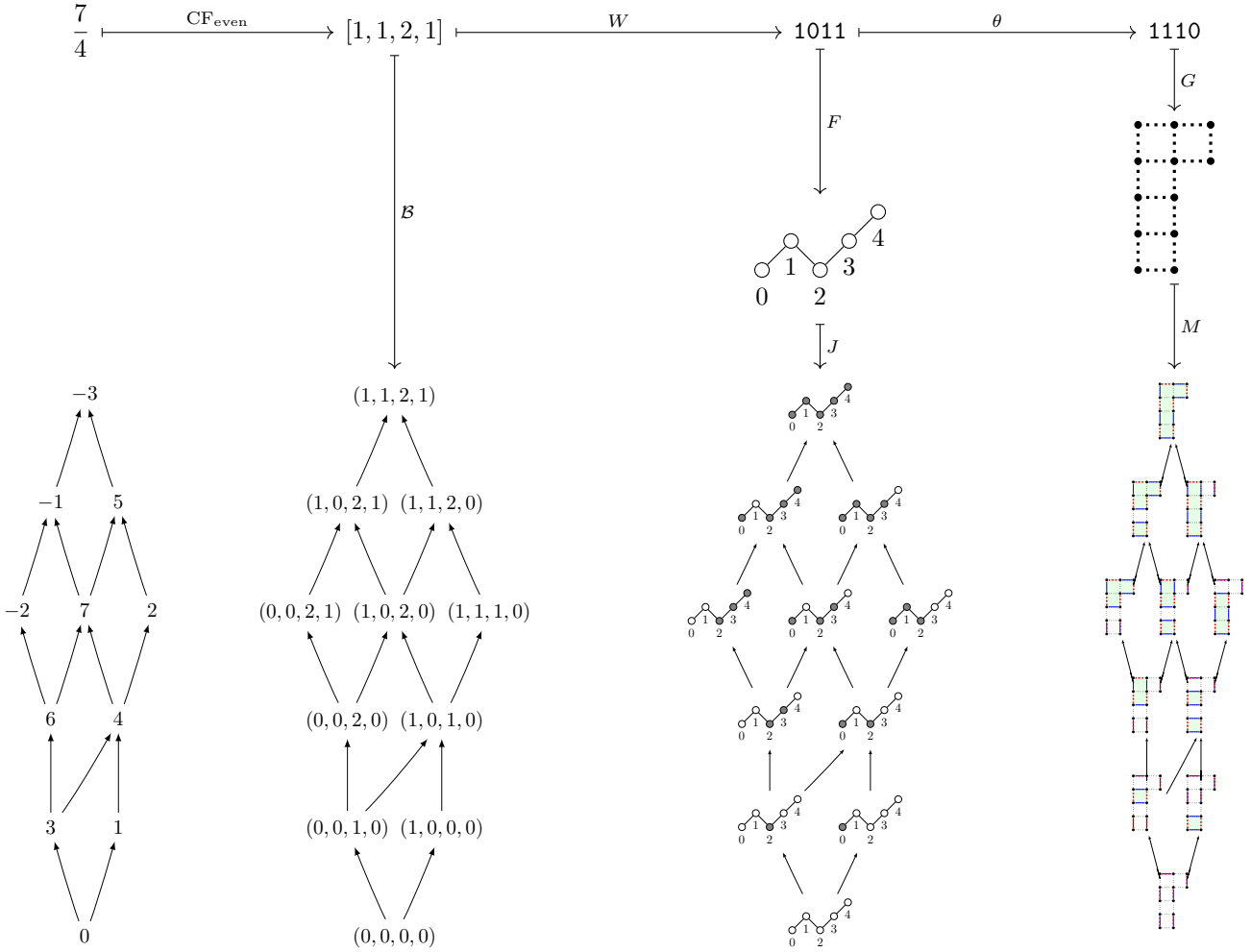
$$\left[\frac{7}{5}\right]_q = q^{-1}\frac{q^5 + 2q^4 + 2q^3 + q^2 + q}{q^3 + 2q^2 + q + 1}$$

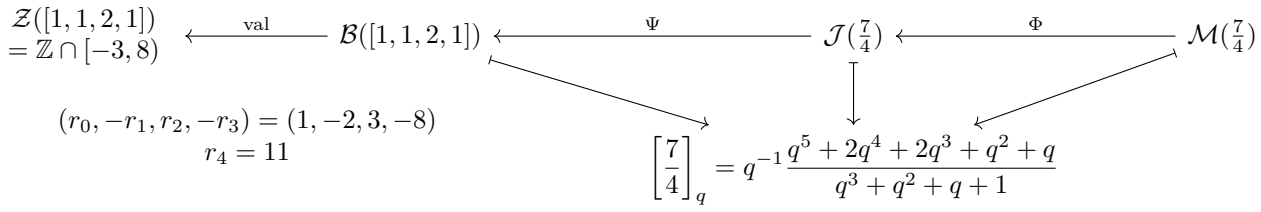


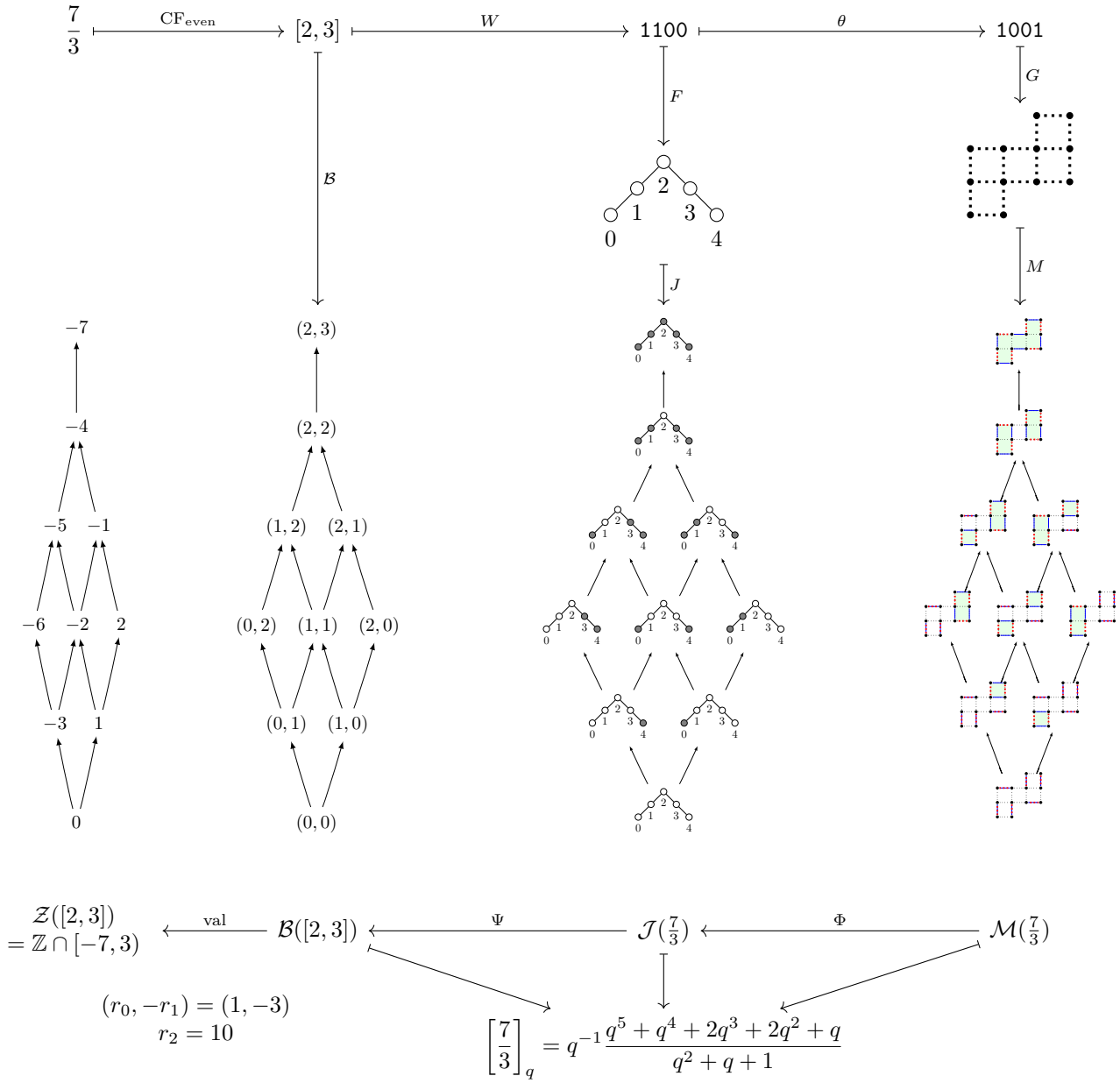
$\mathcal{Z}([1,1,1,2])$
$= \mathbb{Z} \cap [-8,5)$
$\xleftarrow{\mathrm{val}}$
$\mathcal{B}([1,1,1,2])$
$\xleftarrow{\Psi}$
$\mathcal{J}(\frac{8}{5})$
$\xleftarrow{\Phi}$
$\mathcal{M}(\frac{8}{5})$

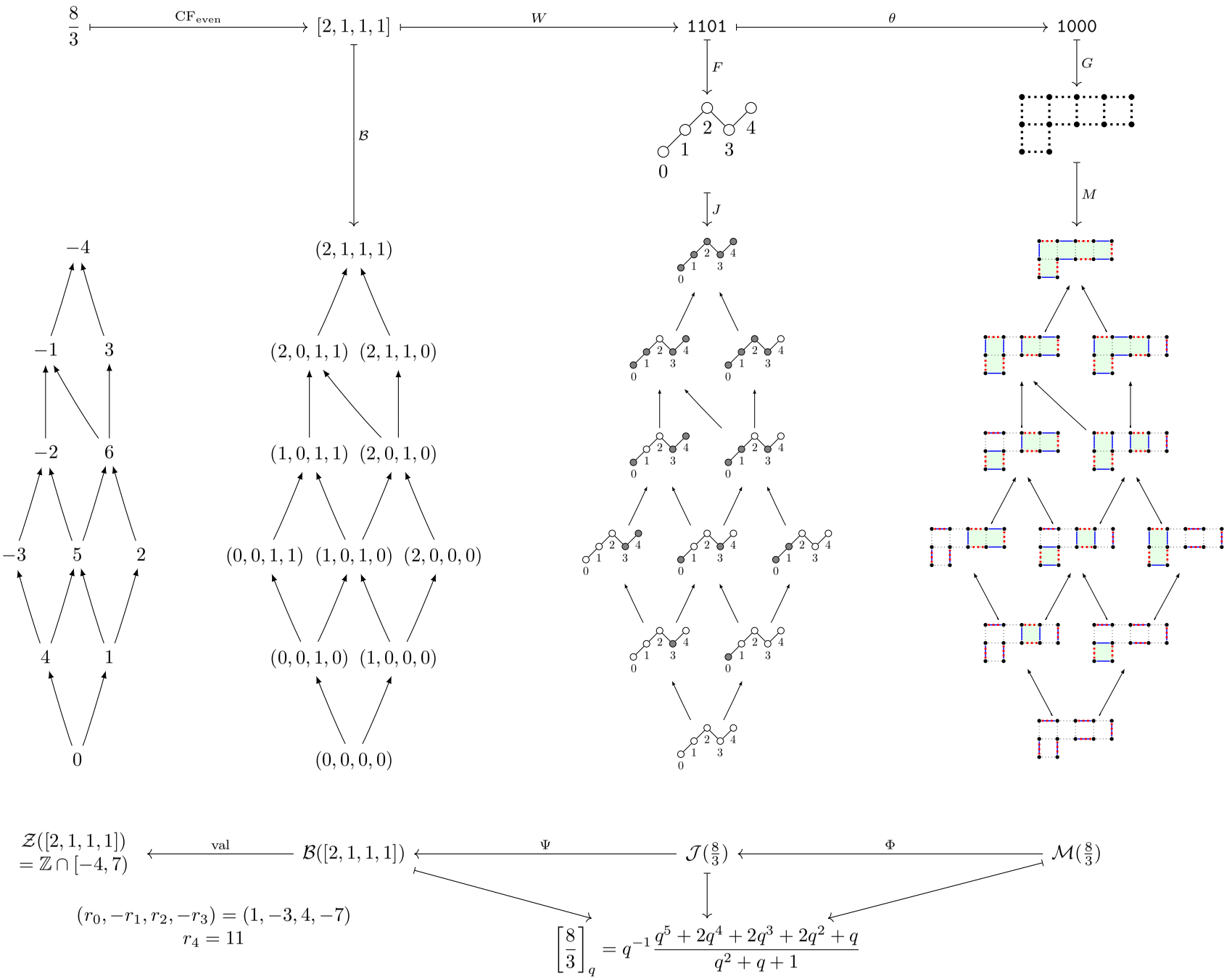
$(r_0, -r_1, r_2, -r_3) = (1, -2, 3, -5)$
$r_4 = 13$

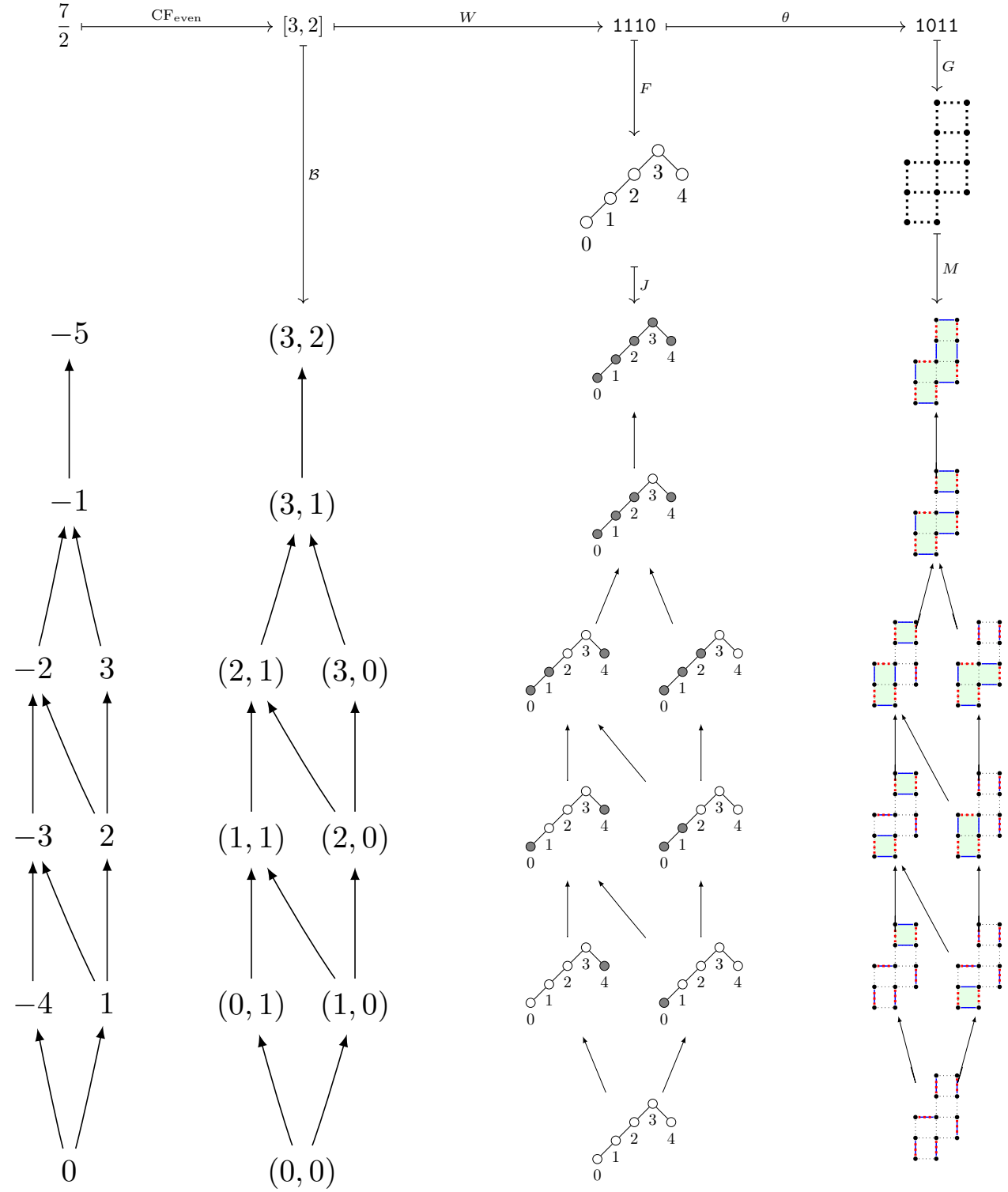
$$\left[\frac{8}{5}\right]_q = q^{-1}\frac{q^5 + 2q^4 + 2q^3 + 2q^2 + q}{q^3 + q^2 + 2q + 1}$$


$\frac{7}{4}$
$\mathrm{CF}_{\mathrm{even}}$
$[1,1,2,1]$
W
1011
θ
1110
F
G
$\mathcal{B}$
J
M
-3
-1 5
-2 7 2
6 4
3 1
0
$(1,1,2,1)$
$(1,0,2,1)$ $(1,1,2,0)$
$(0,0,2,1)$ $(1,0,2,0)$ $(1,1,1,0)$
$(0,0,2,0)$ $(1,0,1,0)$
$(0,0,1,0)$ $(1,0,0,0)$
$(0,0,0,0)$


$\mathcal{Z}([1,1,2,1])$
$= \mathbb{Z} \cap [-3,8)$
val
$\mathcal{B}([1,1,2,1])$
Ψ
$\mathcal{J}(\frac{7}{4})$
Φ
$\mathcal{M}(\frac{7}{4})$
$(r_0, -r_1, r_2, -r_3) = (1,-2,3,-8)$
$r_4 = 11$
$\left[\frac{7}{4}\right]_q = q^{-1}\frac{q^5+2q^4+2q^3+q^2+q}{q^3+q^2+q+1}$


7/3
CF_even
[2, 3]
W
1100
θ
1001
B
F
G
M
J
−7
−4
−5 −1
−6 −2 2
−3 1
0
(2, 3)
(2, 2)
(1, 2) (2, 1)
(0, 2) (1, 1) (2, 0)
(0, 1) (1, 0)
(0, 0)
Z([2, 3])
= Z ∩ [−7, 3)
val
B([2, 3])
Ψ
J(7/3)
Φ
M(7/3)
(r0, −r1) = (1, −3)
r2 = 10
[7/3]q = q^−1 (q^5 + q^4 + 2q^3 + 2q^2 + q)/(q^2 + q + 1)

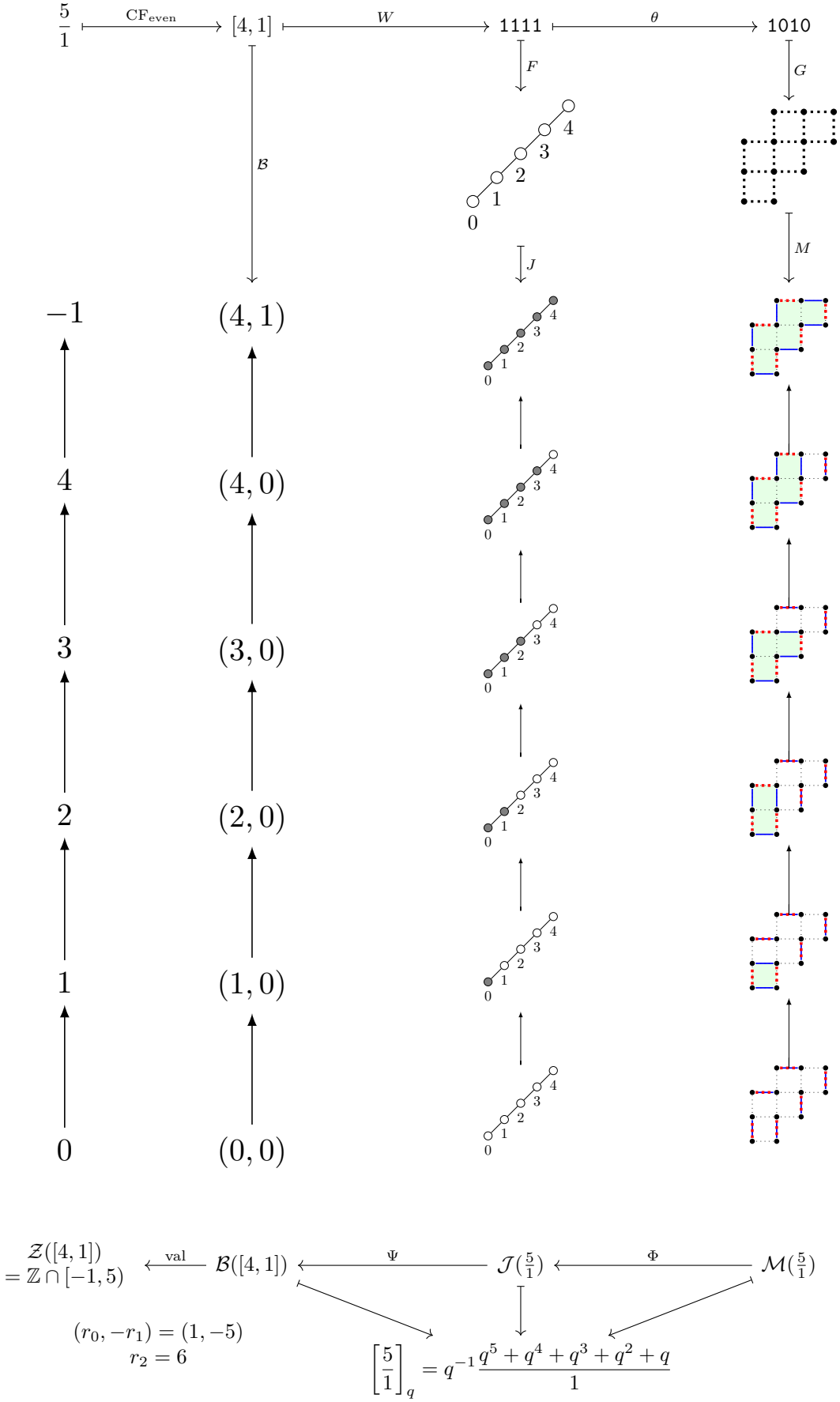

$\frac{8}{3}$
$\mathrm{CF}_{\mathrm{even}}$
$[2,1,1,1]$
W
1101
θ
1000
$\mathcal{B}$
F
G
J
M
-4
-1
3
-2
6
-3
5
2
4
1
0
$(2,1,1,1)$
$(2,0,1,1)$
$(2,1,1,0)$
$(1,0,1,1)$
$(2,0,1,0)$
$(0,0,1,1)$
$(1,0,1,0)$
$(2,0,0,0)$
$(0,0,1,0)$
$(1,0,0,0)$
$(0,0,0,0)$
$\mathcal{Z}([2,1,1,1])$
$= \mathbb{Z} \cap [-4,7)$
val
$\mathcal{B}([2,1,1,1])$
Ψ
$\mathcal{J}(\frac{8}{3})$
Φ
$\mathcal{M}(\frac{8}{3})$
$(r_0, -r_1, r_2, -r_3) = (1,-3,4,-7)$
$r_4 = 11$
$\left[\frac{8}{3}\right]_q = q^{-1}\frac{q^5+2q^4+2q^3+2q^2+q}{q^2+q+1}$



$$\mathcal{Z}([3,2]) = \mathbb{Z} \cap [-5,4) \xleftarrow{\text{val}} \mathcal{B}([3,2]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{7}{2}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{7}{2})$$

$$(r_0, -r_1) = (1,-4)$$
$$r_2 = 9$$

$$\left[\frac{7}{2}\right]_q = q^{-1}\frac{q^5+q^4+2q^3+2q^2+q}{q+1}$$



$$\underset{=\mathbb{Z}\cap[-1,5)}{\mathcal{Z}([4,1])} \xleftarrow{\text{val}} \mathcal{B}([4,1]) \xleftarrow{\Psi} \mathcal{J}(\tfrac{5}{1}) \xleftarrow{\Phi} \mathcal{M}(\tfrac{5}{1})$$

$$(r_0, -r_1) = (1, -5)$$
$$r_2 = 6$$

$$\left[\frac{5}{1}\right]_q = q^{-1}\frac{q^5+q^4+q^3+q^2+q}{1}$$