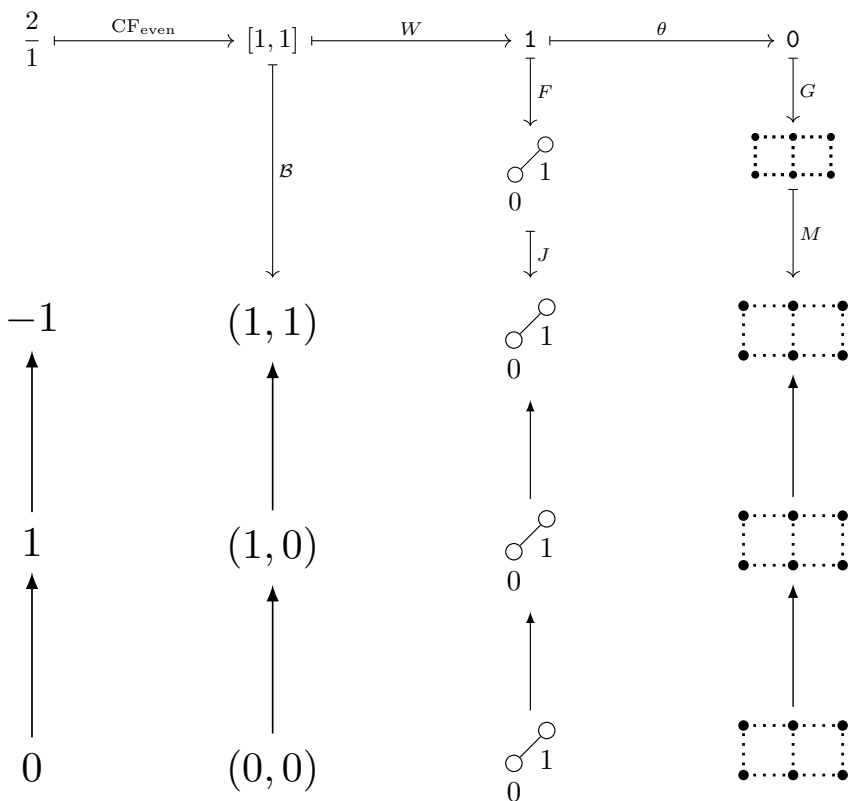
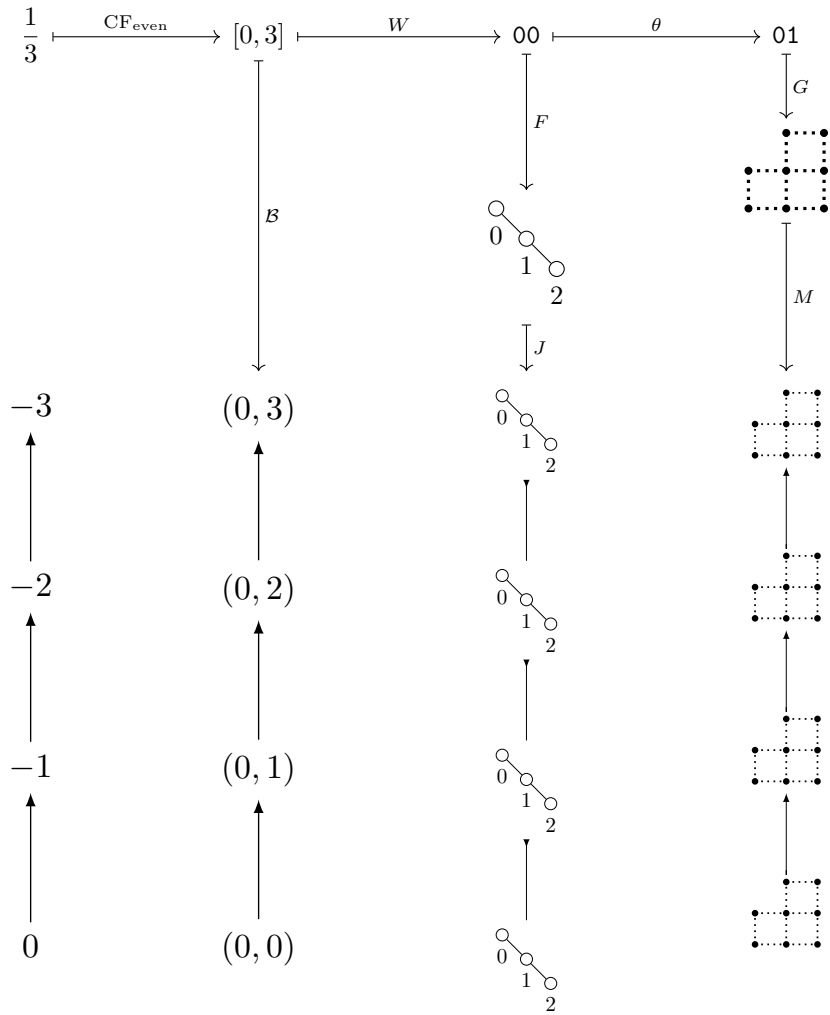


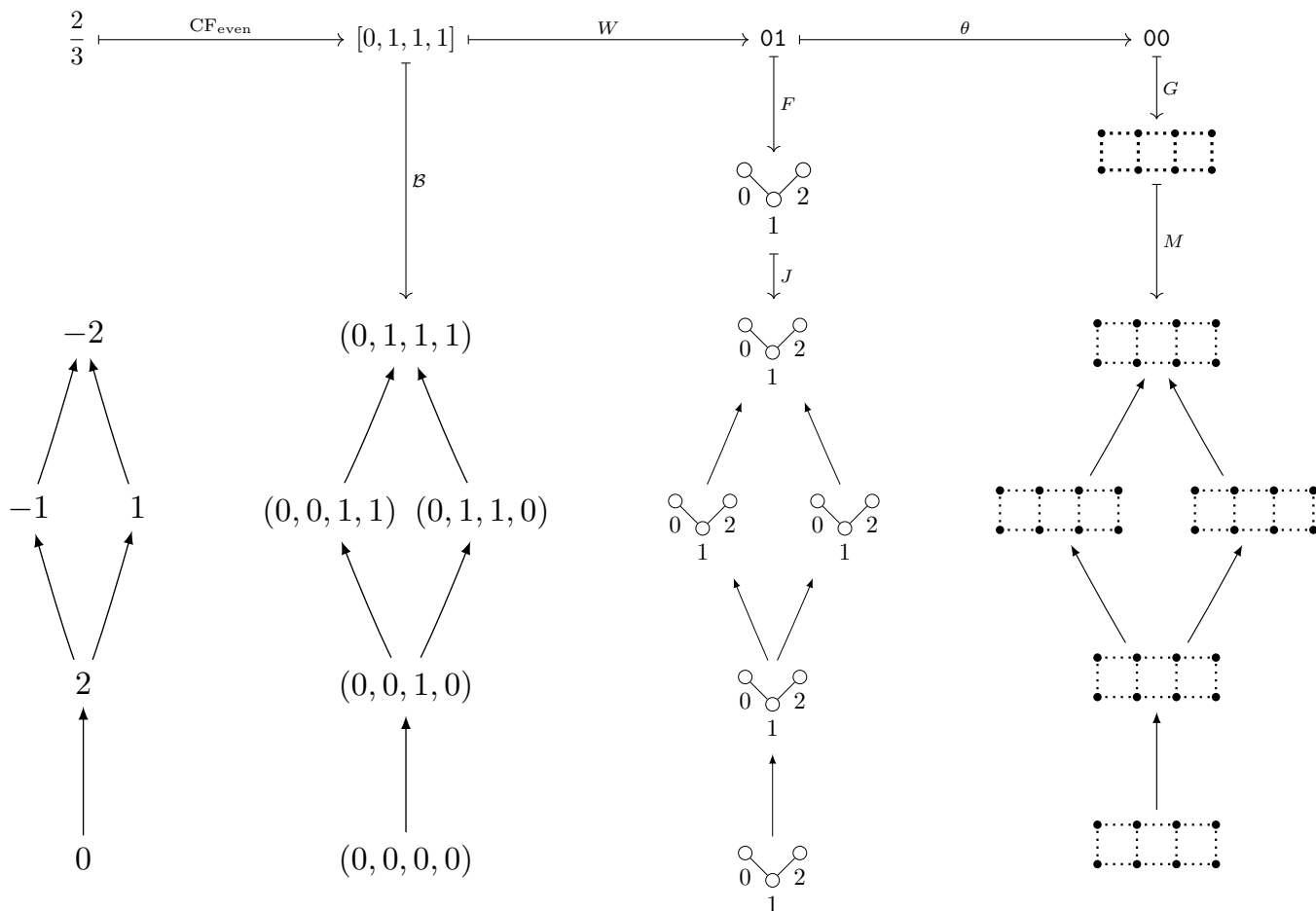
$$\begin{array}{ccccc}
 \mathcal{Z}([0, 2]) & \xleftarrow{\text{val}} & \mathcal{B}([0, 2]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{1}{2}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{1}{2}) \\
 = \mathbb{Z} \cap [-2, 1) & & & & \downarrow & & \swarrow \\
 (r_0, -r_1) = (1, -1) & & & & \downarrow & & \swarrow \\
 r_2 = 3 & & & & \left[\begin{array}{c} 1 \\ 2 \end{array} \right]_q & = & q^{-1} \frac{q^2}{q+1}
 \end{array}$$



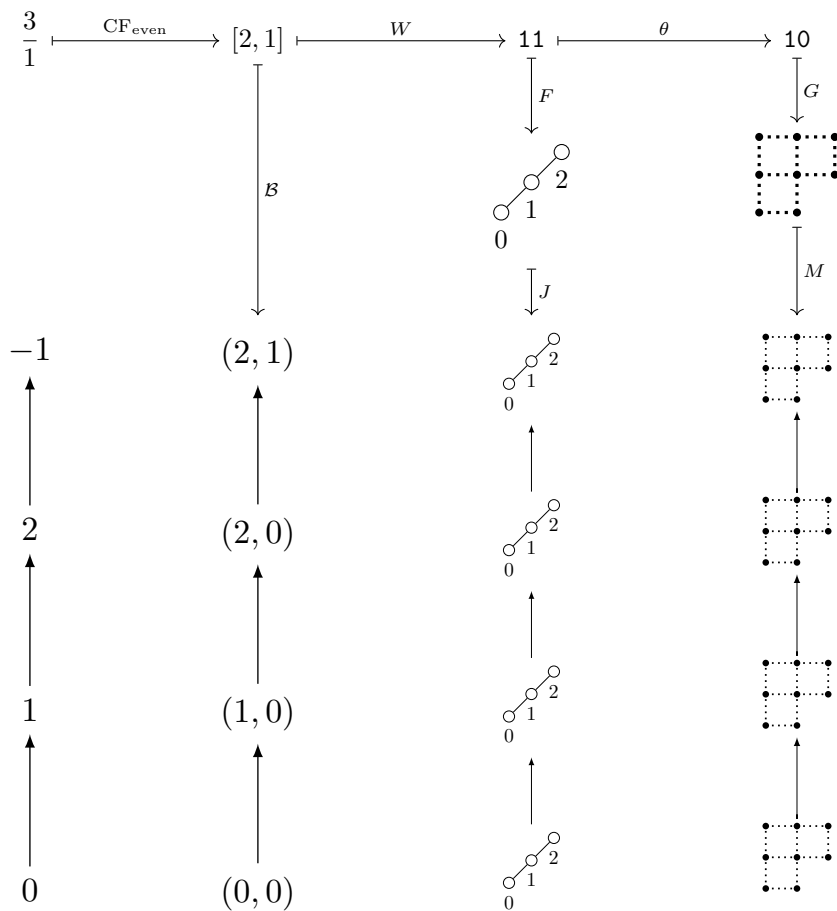
$$\begin{array}{ccccc}
 \mathcal{Z}([1, 1]) & \xleftarrow{\text{val}} & \mathcal{B}([1, 1]) & \xleftarrow{\Psi} & \mathcal{J}\left(\frac{2}{1}\right) & \xleftarrow{\Phi} & \mathcal{M}\left(\frac{2}{1}\right) \\
 = \mathbb{Z} \cap [-1, 2) & & & & \downarrow & & \swarrow \\
 (r_0, -r_1) = (1, -2) & & & & \left[\begin{array}{c} 2 \\ \frac{2}{1} \end{array} \right]_q & = & q^{-1} \frac{q^2 + q}{1} \\
 r_2 = 3 & & & & & &
 \end{array}$$



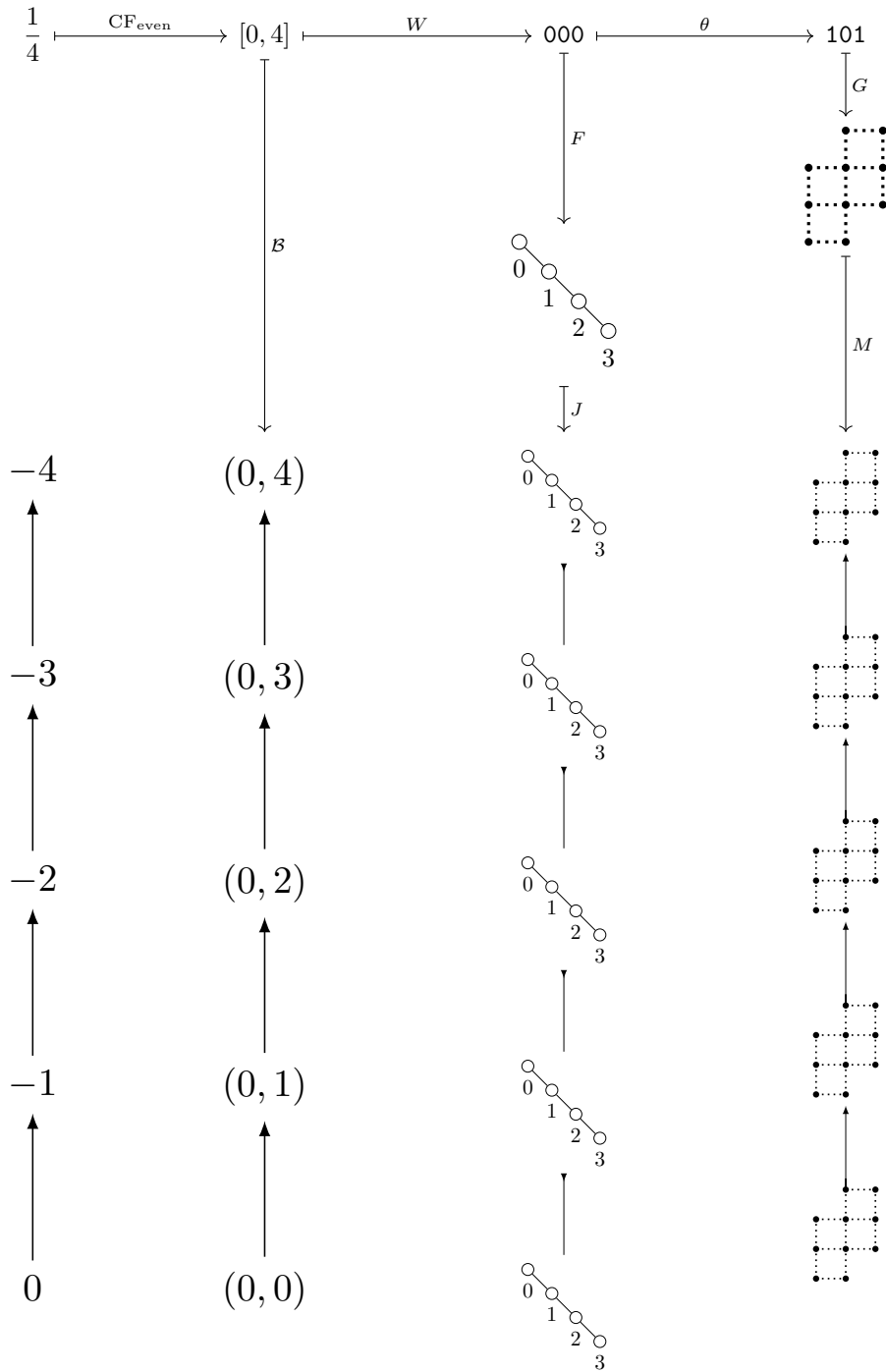
$$\begin{array}{ccccc}
 \mathcal{Z}([0, 3]) & \xleftarrow{\text{val}} & \mathcal{B}([0, 3]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{1}{3}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{1}{3}) \\
 = \mathbb{Z} \cap [-3, 1) & & & & \downarrow & & \swarrow \\
 (r_0, -r_1) = (1, -1) & & & & \left[\frac{1}{3} \right]_q & = & q^{-1} \frac{q^3}{q^2 + q + 1} \\
 r_2 = 4 & & & & & &
 \end{array}$$



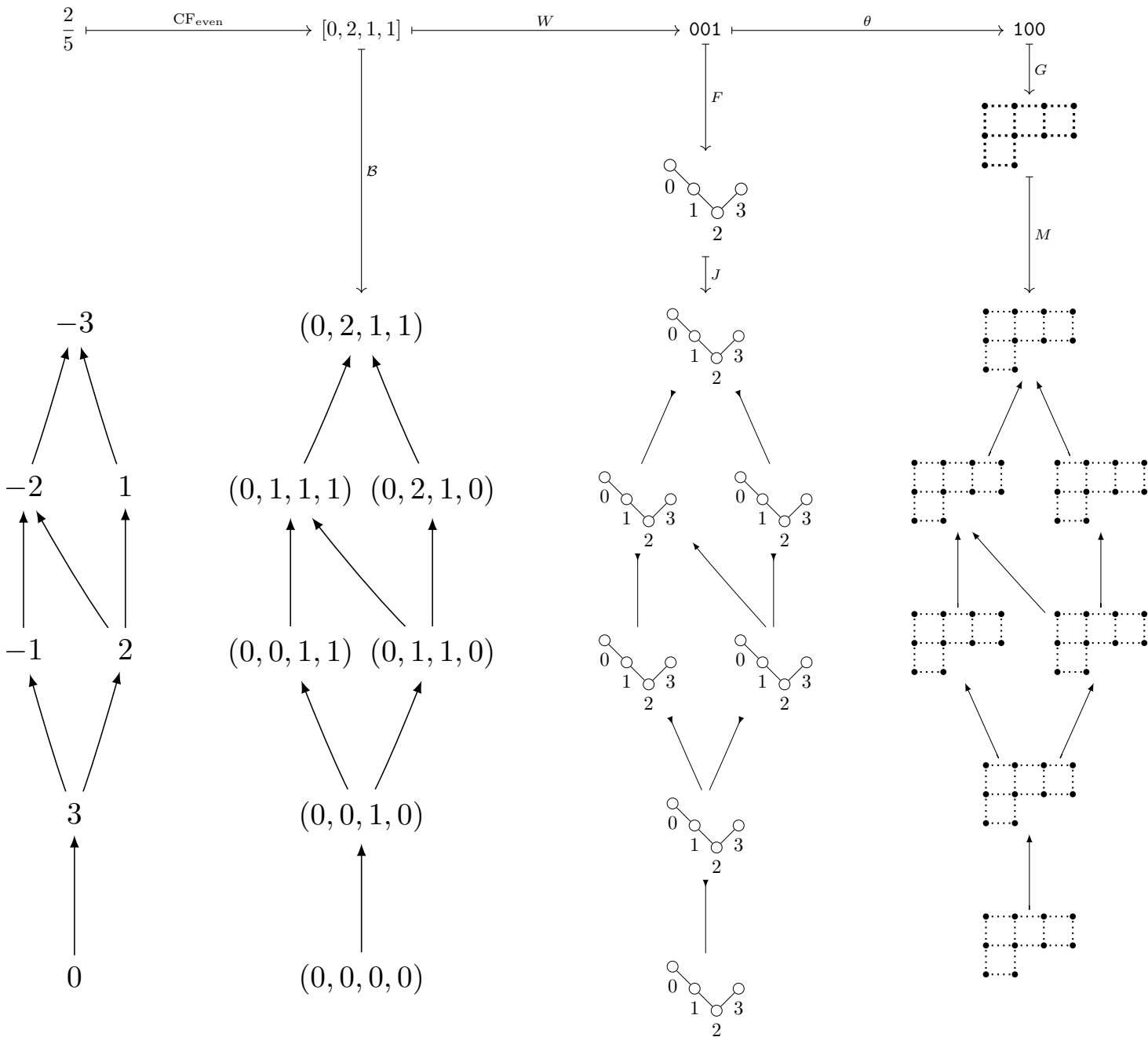
$$\begin{array}{c}
 \mathcal{Z}([0, 1, 1, 1]) \xleftarrow{\text{val}} \mathcal{B}([0, 1, 1, 1]) \xleftarrow{\Psi} \mathcal{J}(\frac{2}{3}) \xleftarrow{\Phi} \mathcal{M}(\frac{2}{3}) \\
 = \mathbb{Z} \cap [-2, 3] \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -3) \\
 r_4 = 5 \\
 \left[\frac{2}{3} \right]_q = q^{-1} \frac{q^3 + q^2}{q^2 + q + 1}
 \end{array}$$



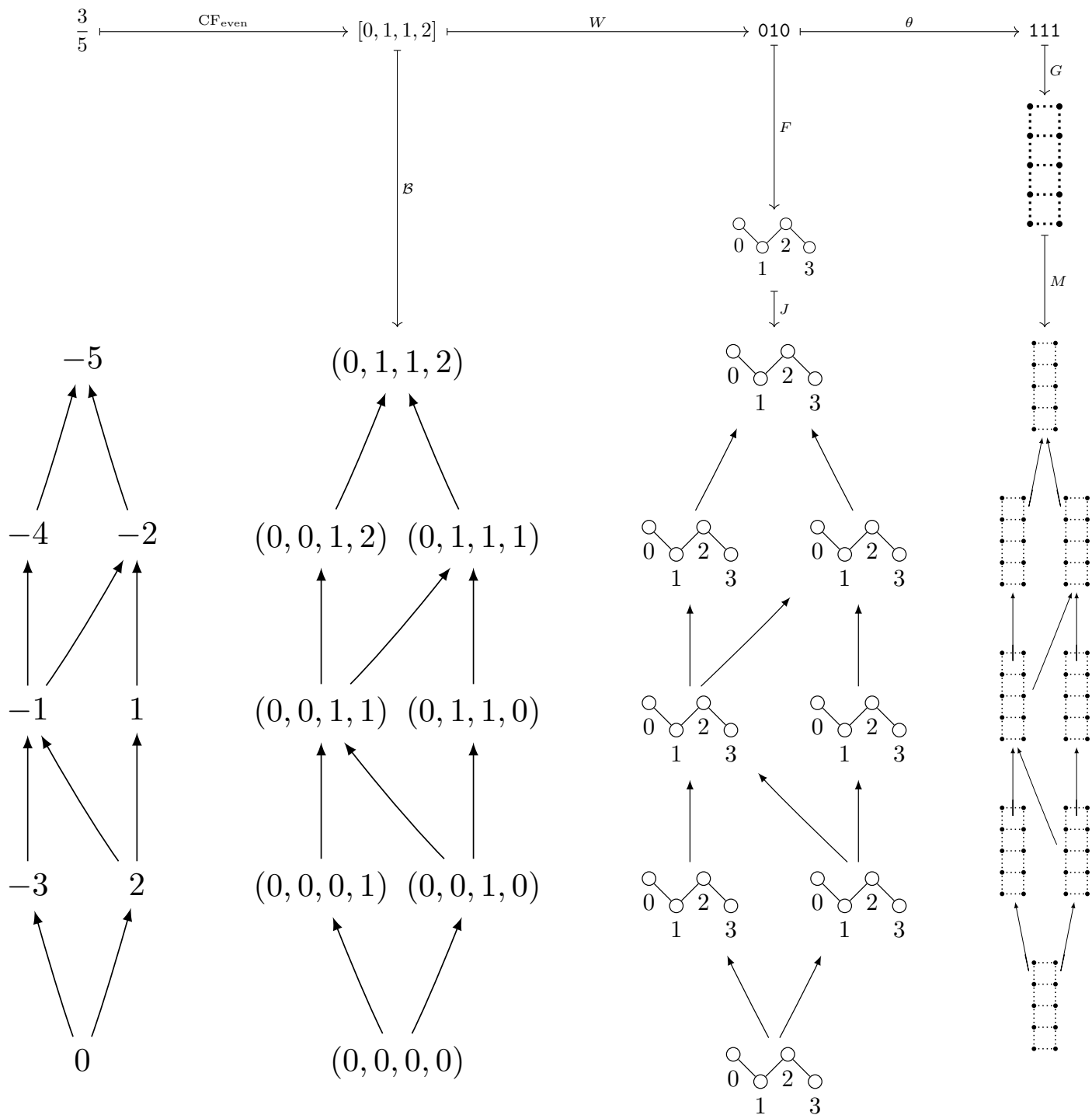
$$\begin{array}{ccccc}
 \mathcal{Z}([2, 1]) & \xleftarrow{\text{val}} & \mathcal{B}([2, 1]) & \xleftarrow{\Psi} & \mathcal{J}\left(\frac{3}{1}\right) & \xleftarrow{\Phi} & \mathcal{M}\left(\frac{3}{1}\right) \\
 = \mathbb{Z} \cap [-1, 3) & & & & \downarrow & & \downarrow \\
 (r_0, -r_1) = (1, -3) & & & & \left[\begin{array}{c} 3 \\ -1 \end{array} \right]_q & = & q^{-1} \frac{q^3 + q^2 + q}{1} \\
 r_2 = 4 & & & & & &
 \end{array}$$



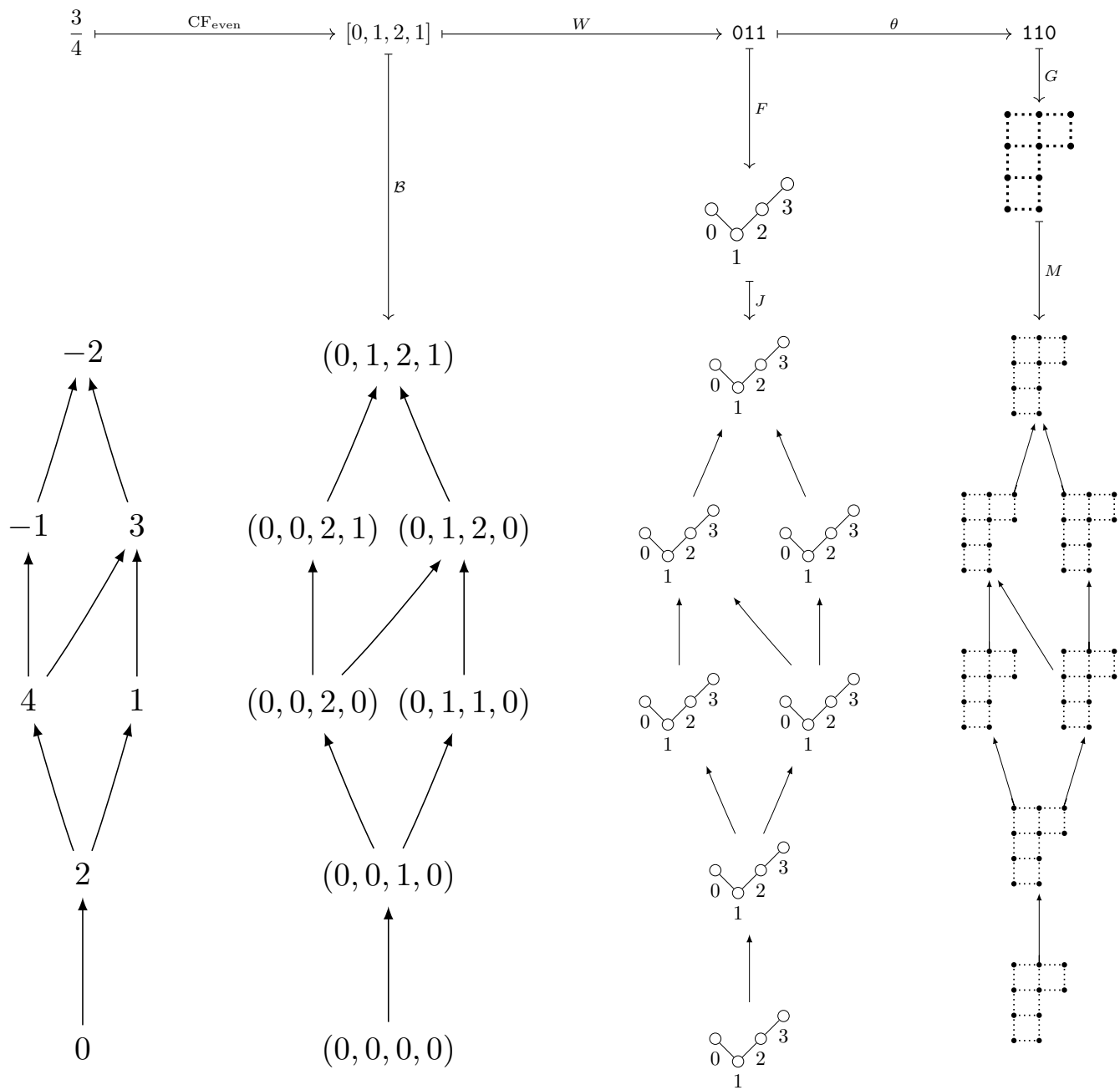
$$\begin{array}{ccccc}
 \mathcal{Z}([0, 4]) & \xleftarrow{\text{val}} & \mathcal{B}([0, 4]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{1}{4}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{1}{4}) \\
 = \mathbb{Z} \cap [-4, 1) & & & & \downarrow & & \downarrow \\
 (r_0, -r_1) = (1, -1) & & & & \downarrow & & \downarrow \\
 r_2 = 5 & & & & \left[\frac{1}{4} \right]_q & = & q^{-1} \frac{q^4}{q^3 + q^2 + q + 1}
 \end{array}$$



$$\begin{array}{c}
 \mathcal{Z}([0, 2, 1, 1]) \leftarrow \text{val} \mathcal{B}([0, 2, 1, 1]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{2}{5}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{2}{5}\right) \\
 = \mathbb{Z} \cap [-3, 4) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 3, -4) \\
 r_4 = 7 \\
 \left[\begin{array}{c} 2 \\ 5 \end{array} \right]_q = q^{-1} \frac{q^4 + q^3}{q^3 + 2q^2 + q + 1}
 \end{array}$$

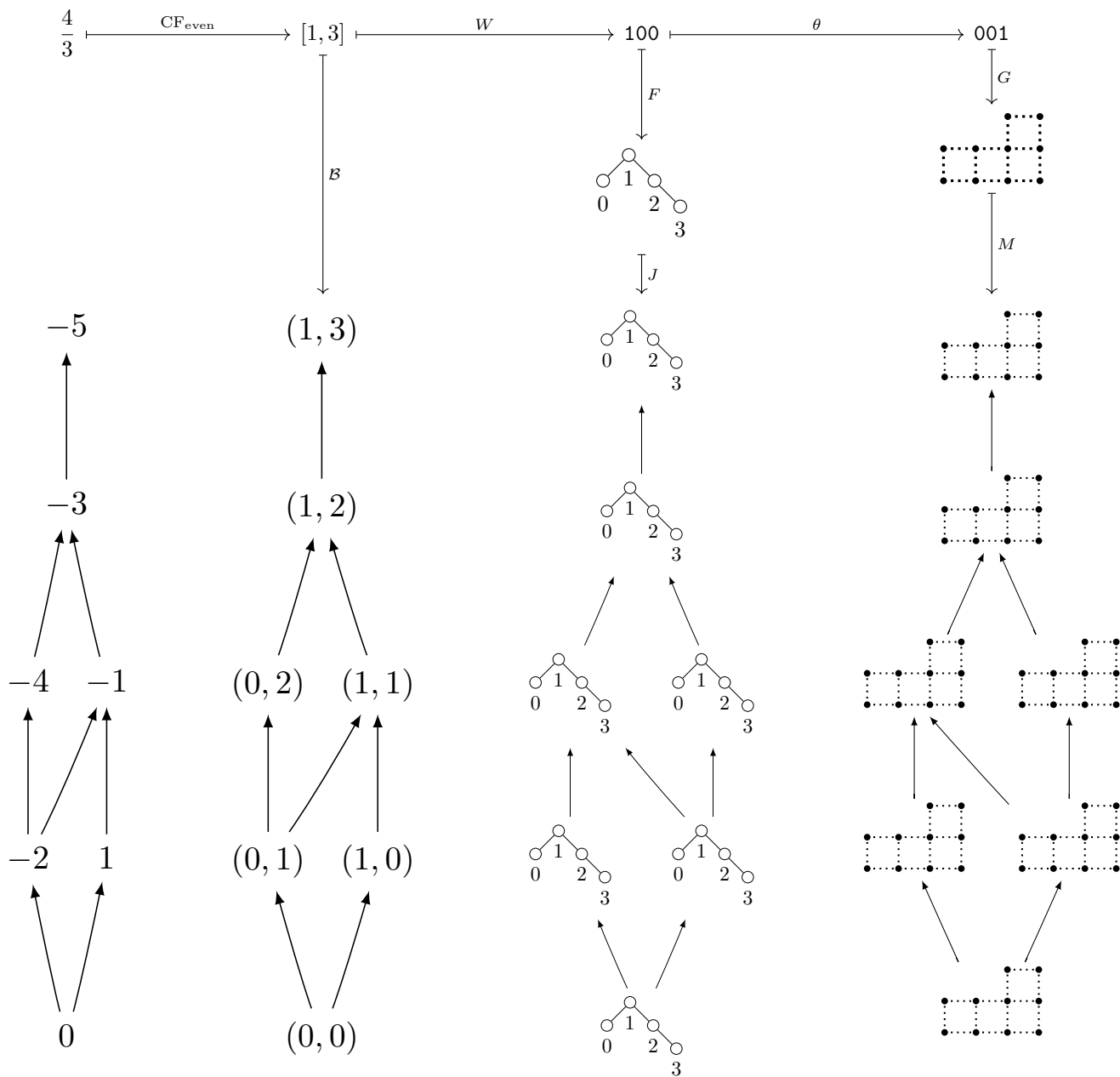


$$\begin{array}{c}
 \mathcal{Z}([0, 1, 1, 2]) \xleftarrow{\text{val}} \mathcal{B}([0, 1, 1, 2]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{3}{5}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{3}{5}\right) \\
 = \mathbb{Z} \cap [-5, 3) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -3) \\
 r_4 = 8 \\
 \left[\begin{array}{c} 3 \\ 5 \end{array} \right]_q = q^{-1} \frac{q^4 + q^3 + q^2}{q^3 + q^2 + 2q + 1}
 \end{array}$$

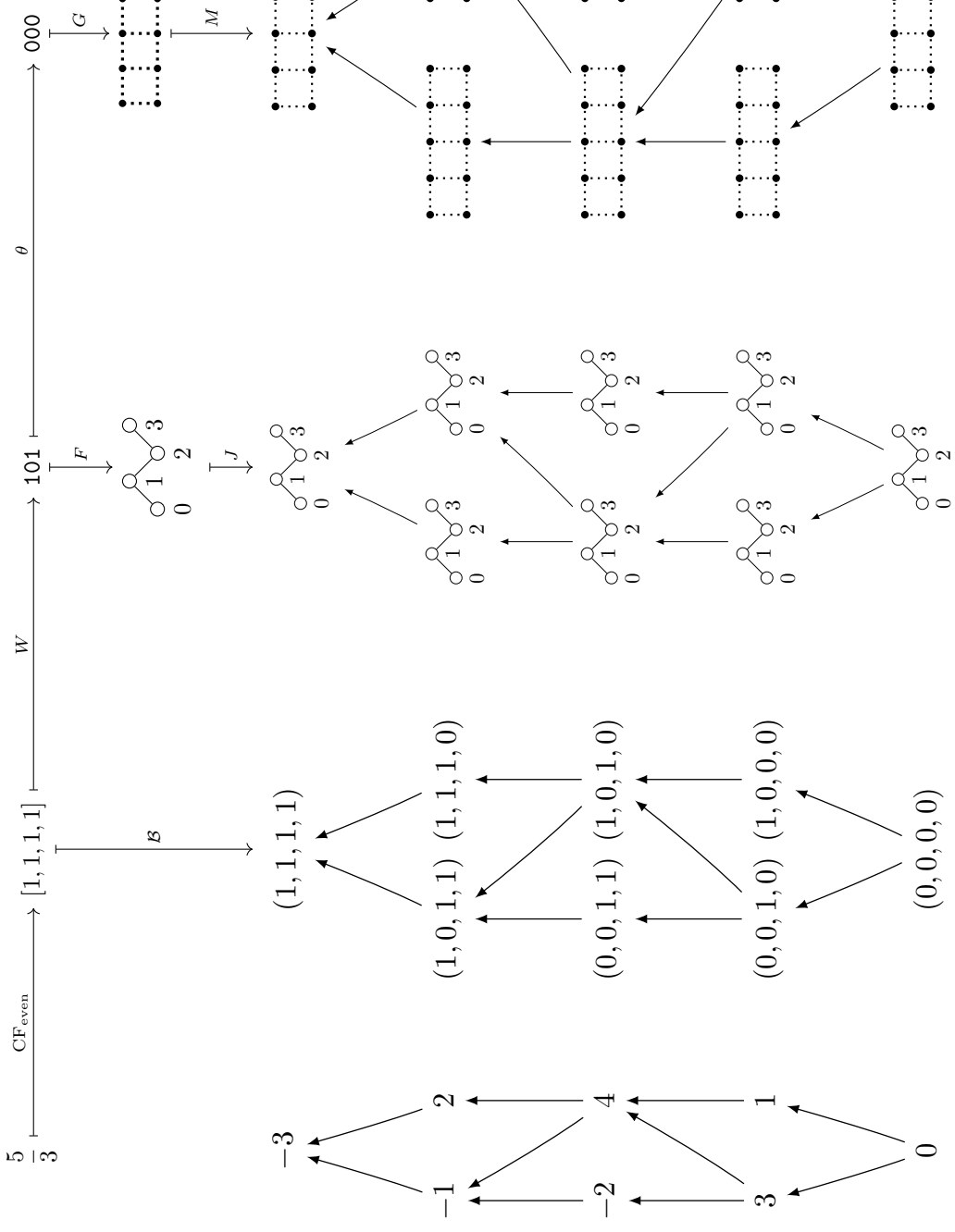


$$\begin{array}{c}
 \mathcal{Z}([0, 1, 2, 1]) \\
 = \mathbb{Z} \cap [-2, 5) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -5) \\
 r_4 = 7
 \end{array}
 \xleftarrow{\text{val}}
 \mathcal{B}([0, 1, 2, 1])
 \xleftarrow{\Psi}
 \mathcal{J}\left(\frac{3}{4}\right)
 \xleftarrow{\Phi}
 \mathcal{M}\left(\frac{3}{4}\right)$$

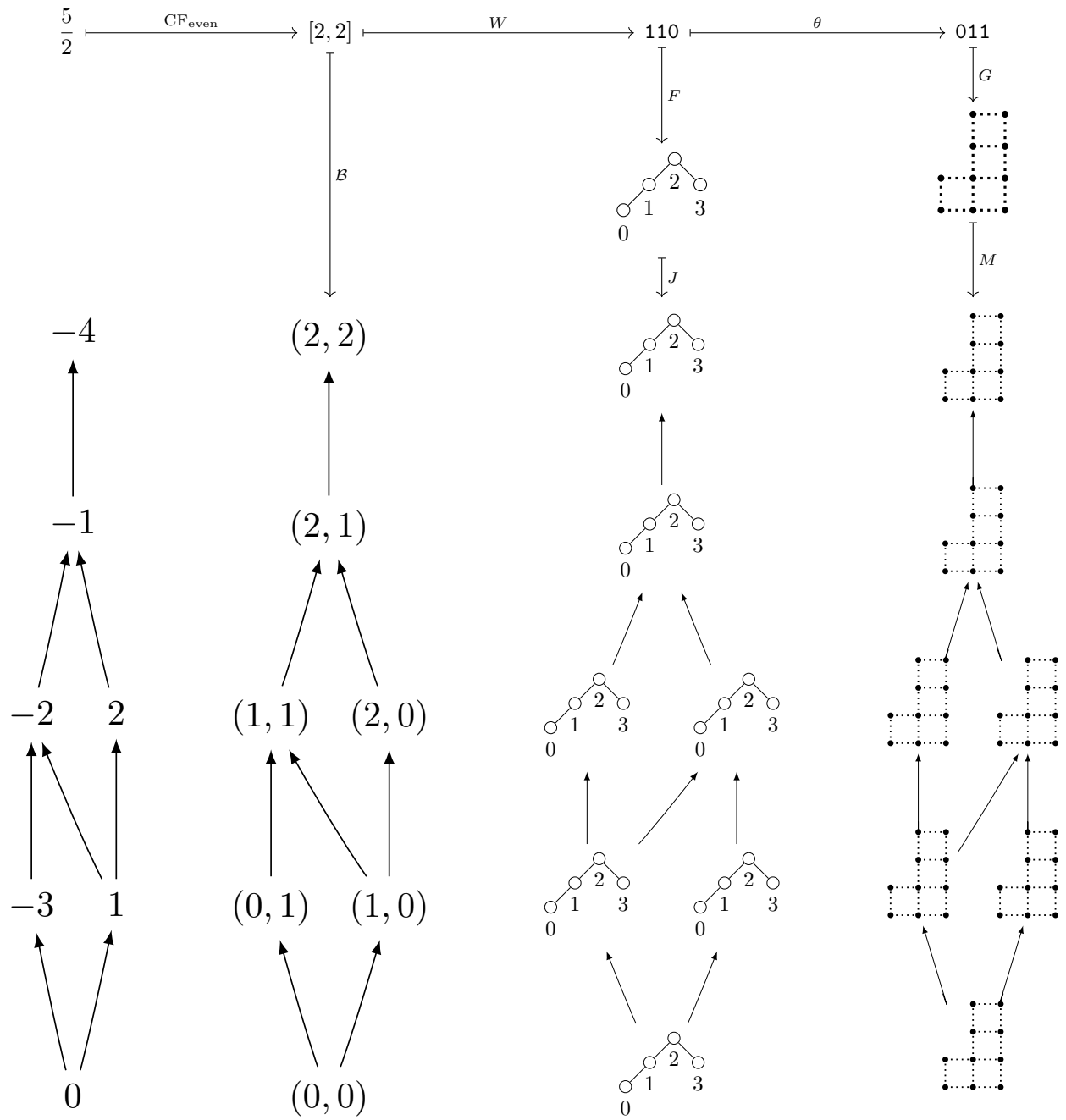
$$\begin{array}{c}
 \mathcal{J}\left(\frac{3}{4}\right) \\
 \downarrow \\
 \left[\begin{array}{c} 3 \\ -4 \end{array} \right]_q = q^{-1} \frac{q^4 + q^3 + q^2}{q^3 + q^2 + q + 1}
 \end{array}$$



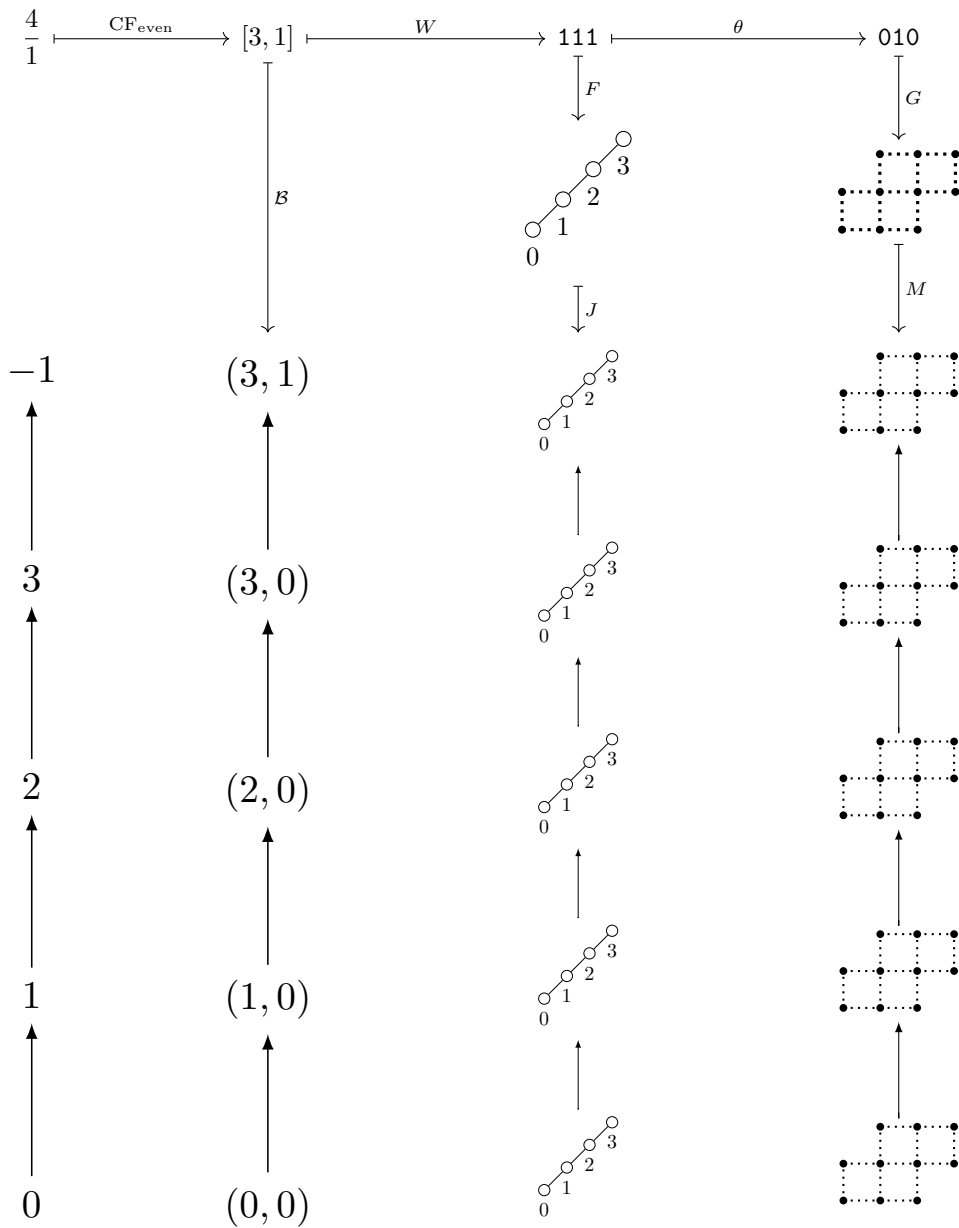
$$\begin{array}{c}
 \mathcal{Z}([1, 3]) \\
 = \mathbb{Z} \cap [-5, 2) \\
 \xleftarrow{\text{val}} \mathcal{B}([1, 3]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{4}{3}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{4}{3}\right) \\
 \begin{array}{l}
 (r_0, -r_1) = (1, -2) \\
 r_2 = 7
 \end{array} \\
 \downarrow \quad \downarrow \quad \downarrow \\
 \left[\frac{4}{3} \right]_q = q^{-1} \frac{q^4 + q^3 + q^2 + q}{q^2 + q + 1}
 \end{array}$$



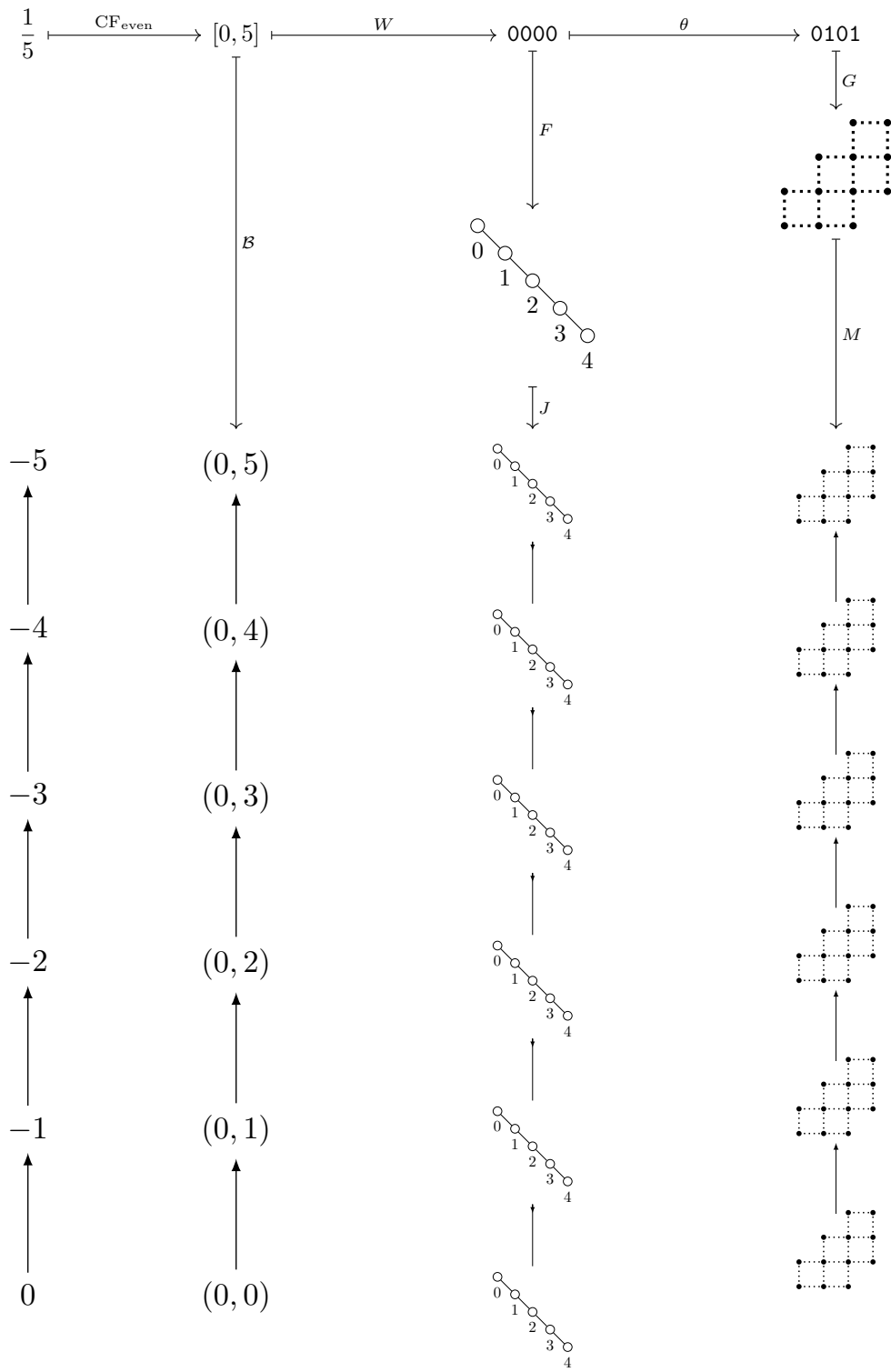
$$\begin{array}{c}
 \mathcal{Z}([1, 1, 1, 1]) \\
 = \mathbb{Z} \cap [-3, 5] \\
 \leftarrow \text{val} \longrightarrow \mathcal{B}([1, 1, 1, 1]) \xleftarrow{\Psi} \mathcal{J}(\frac{5}{3}) \xleftarrow{\Phi} \mathcal{M}(\frac{5}{3}) \\
 \begin{array}{l}
 (r_0, -r_1, r_2, -r_3) = (1, -2, 3, -5) \\
 r_4 = 8
 \end{array} \\
 \begin{array}{l}
 \left[\begin{array}{c} 5 \\ 3 \end{array} \right]_q \\
 = q^{-1} \frac{q^4 + 2q^3 + q^2 + q}{q^2 + q + 1}
 \end{array}
 \end{array}$$



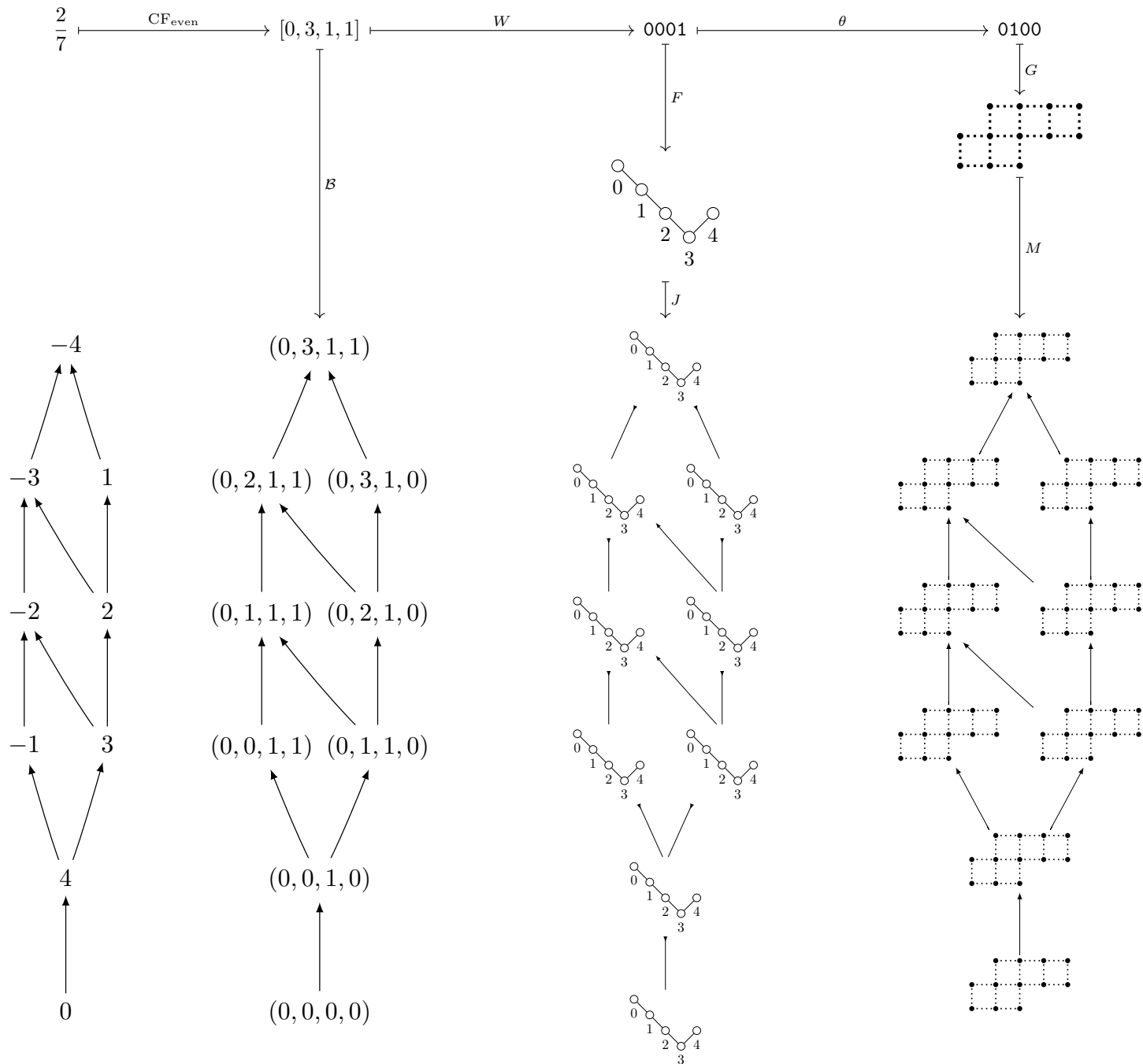
$$\begin{array}{ccccc}
 \mathcal{Z}([2, 2]) & \xleftarrow{\text{val}} & \mathcal{B}([2, 2]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{5}{2}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{5}{2}) \\
 = \mathbb{Z} \cap [-4, 3) & & & & \downarrow & & \\
 (r_0, -r_1) = (1, -3) & & & & \downarrow & & \\
 r_2 = 7 & & & & \left[\frac{5}{2} \right]_q & = & q^{-1} \frac{q^4 + q^3 + 2q^2 + q}{q + 1}
 \end{array}$$



$$\begin{array}{ccccc}
 \mathcal{Z}([3, 1]) & \xleftarrow{\text{val}} & \mathcal{B}([3, 1]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{4}{1}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{4}{1}) \\
 = \mathbb{Z} \cap [-1, 4) & & & & \downarrow & & \swarrow \\
 (r_0, -r_1) = (1, -4) & & & & \downarrow & & \\
 r_2 = 5 & & & & \left[\frac{4}{1} \right]_q = q^{-1} \frac{q^4 + q^3 + q^2 + q}{1} & &
 \end{array}$$

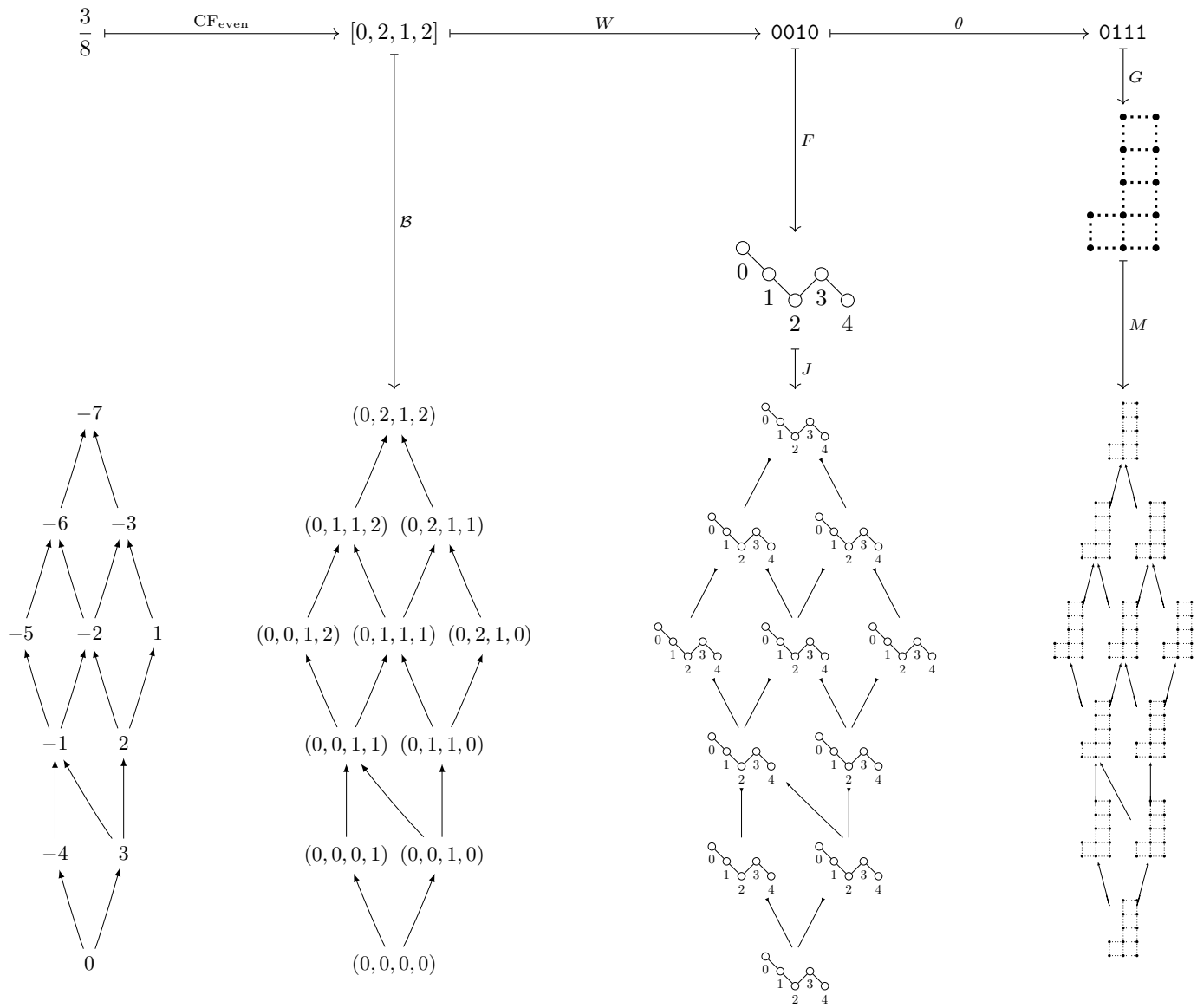


$$\begin{array}{ccccc}
 \mathcal{Z}([0, 5]) & \xleftarrow{\text{val}} & \mathcal{B}([0, 5]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{1}{5}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{1}{5}) \\
 = \mathbb{Z} \cap [-5, 1) & & & & & & \\
 (r_0, -r_1) = (1, -1) & & & & & & \\
 r_2 = 6 & & & & & & \\
 & & & & \downarrow & & \\
 & & & & \left[\frac{1}{5} \right]_q = q^{-1} \frac{q^5}{q^4 + q^3 + q^2 + q + 1} & &
 \end{array}$$

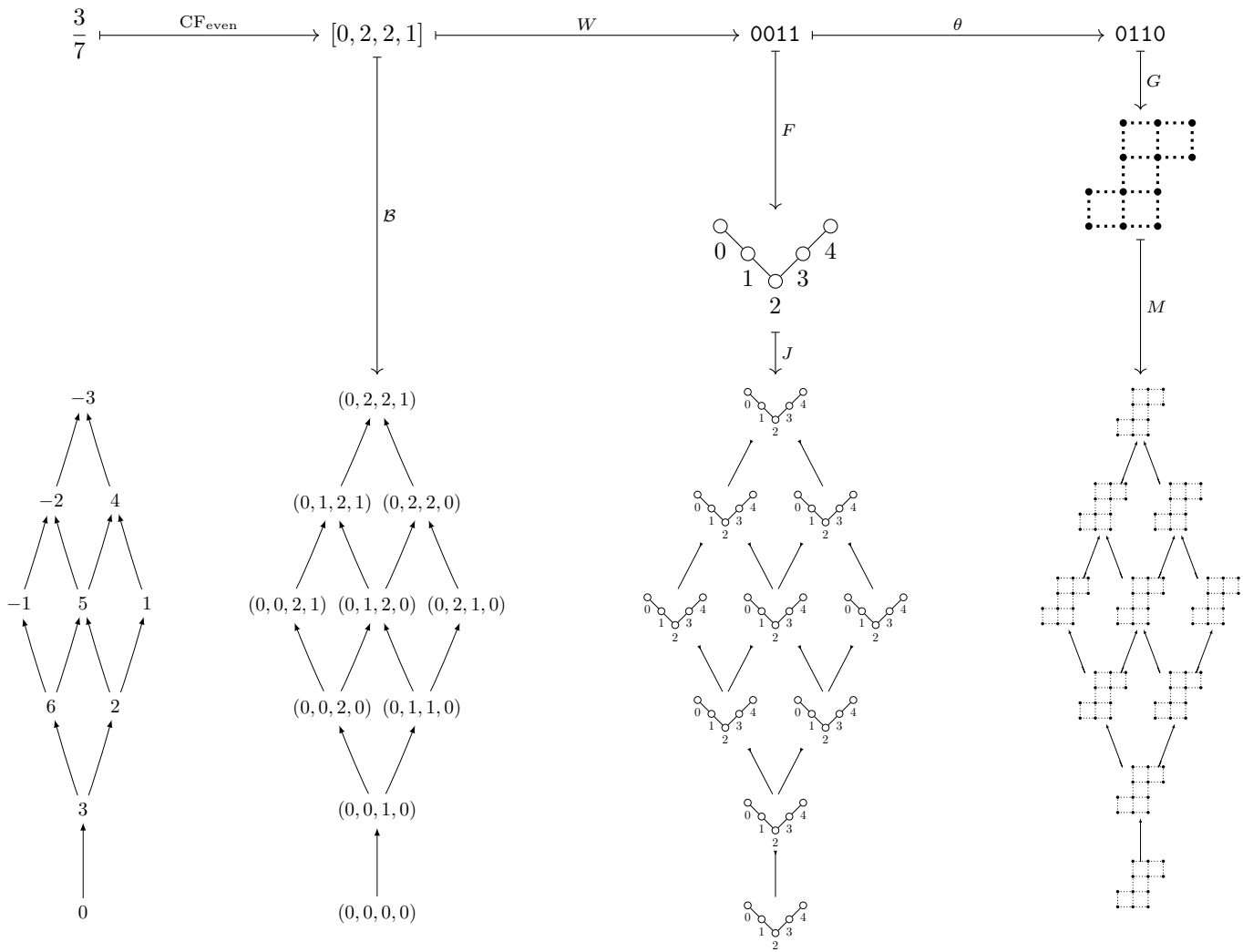


$$\begin{array}{c}
 \mathcal{Z}([0, 3, 1, 1]) \\
 = \mathbb{Z} \cap [-4, 5) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 4, -5) \\
 r_4 = 9
 \end{array}
 \xleftarrow{\text{val}}
 \mathcal{B}([0, 3, 1, 1])
 \xleftarrow{\Psi}
 \mathcal{J}\left(\frac{2}{7}\right)
 \xleftarrow{\Phi}
 \mathcal{M}\left(\frac{2}{7}\right)$$

$$\begin{array}{c}
 \left[\frac{2}{7} \right]_q = q^{-1} \frac{q^5 + q^4}{q^4 + 2q^3 + 2q^2 + q + 1}
 \end{array}$$

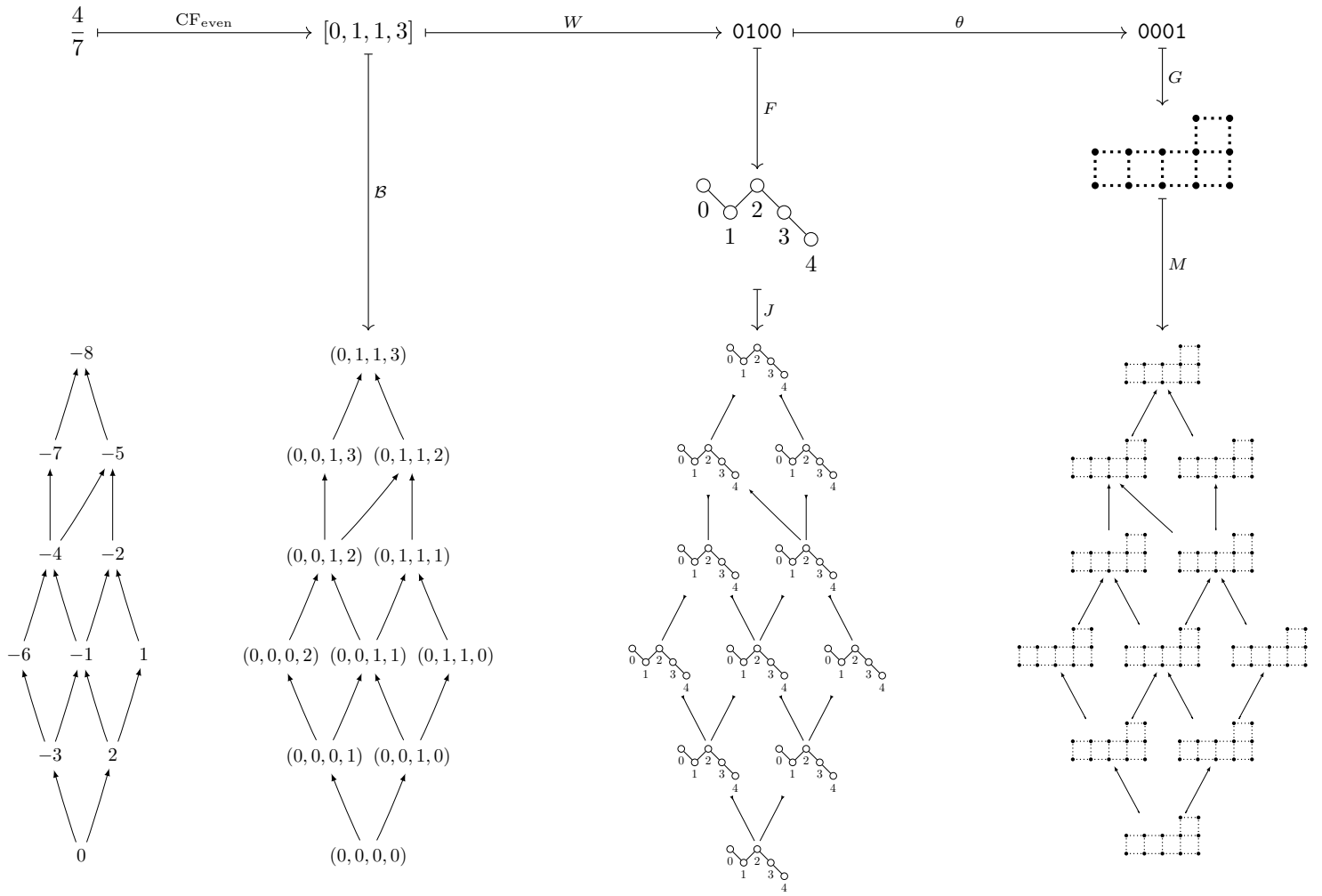


$$\begin{array}{c}
 \mathcal{Z}([0, 2, 1, 2]) \leftarrow \text{val} \mathcal{B}([0, 2, 1, 2]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{3}{8}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{3}{8}\right) \\
 = \mathbb{Z} \cap [-7, 4) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 3, -4) \\
 r_4 = 11 \\
 \left[\frac{3}{8}\right]_q = q^{-1} \frac{q^5 + q^4 + q^3}{q^4 + 2q^3 + 2q^2 + 2q + 1}
 \end{array}$$

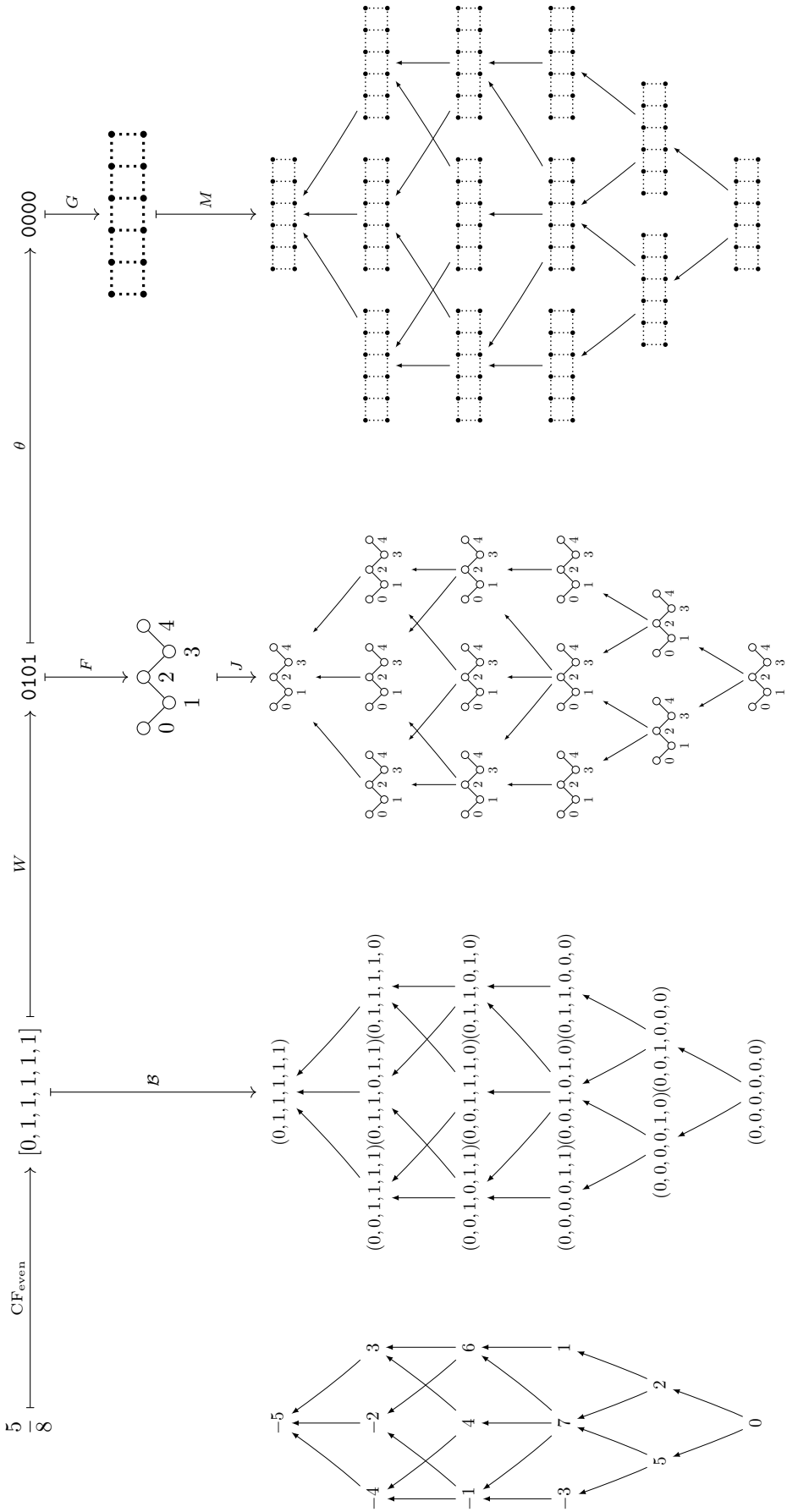


$$\begin{array}{c}
 \mathcal{Z}([0, 2, 2, 1]) \\
 = \mathbb{Z} \cap [-3, 7) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 3, -7) \\
 r_4 = 10
 \end{array}
 \xleftarrow{\text{val}}
 \mathcal{B}([0, 2, 2, 1])
 \xleftarrow{\Psi}
 \mathcal{J}\left(\frac{3}{7}\right)
 \xleftarrow{\Phi}
 \mathcal{M}\left(\frac{3}{7}\right)$$

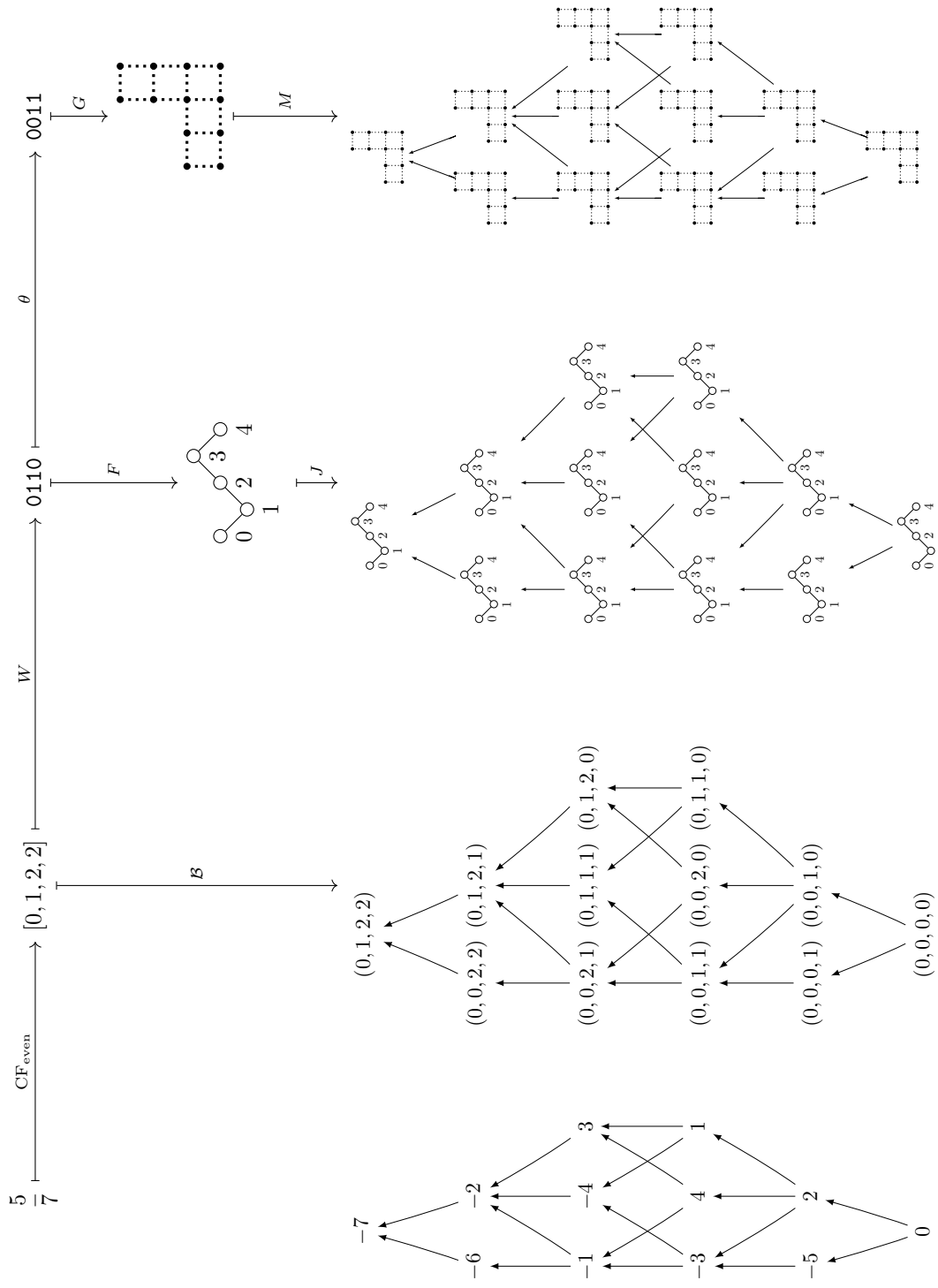
$$\left[\frac{3}{7} \right]_q = q^{-1} \frac{q^5 + q^4 + q^3}{q^4 + 2q^3 + 2q^2 + q + 1}$$



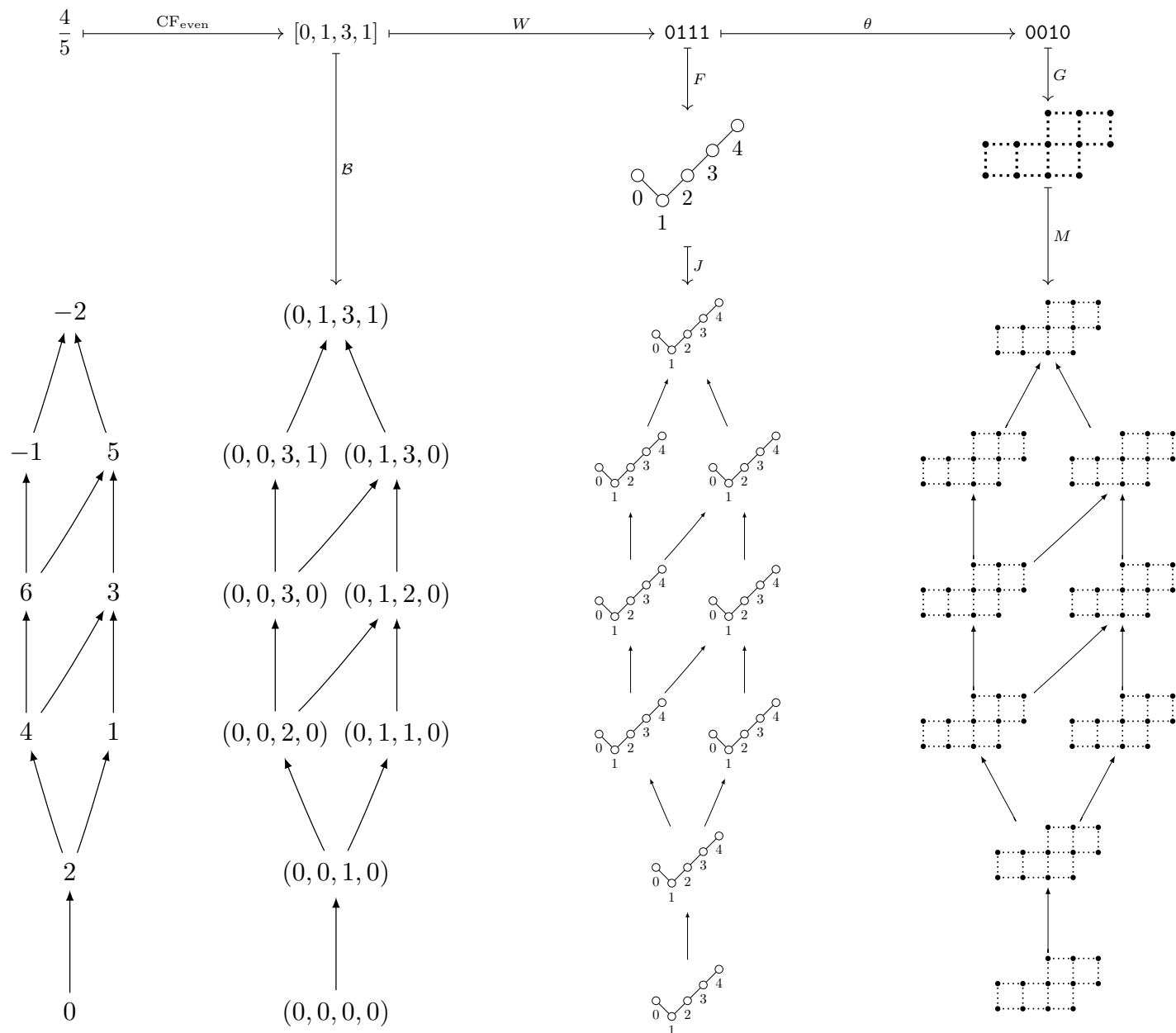
$$\begin{array}{c}
 \mathcal{Z}([0, 1, 1, 3]) \\
 = \mathbb{Z} \cap [-8, 3] \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -3) \\
 r_4 = 11 \\
 \left[\frac{4}{7} \right]_q = q^{-1} \frac{q^5 + q^4 + q^3 + q^2}{q^4 + q^3 + 2q^2 + 2q + 1}
 \end{array}
 \begin{array}{c}
 \xleftarrow{\text{val}} \mathcal{B}([0, 1, 1, 3]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{4}{7}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{4}{7}\right) \\
 \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \\
 \end{array}$$



$$\begin{array}{c}
 \mathcal{Z}([0, 1, 1, 1, 1, 1]) \\
 = \mathbb{Z} \cap [-5, 8] \\
 \begin{array}{l}
 (r_0, -r_1, r_2, -r_3, r_4, -r_5) = (1, -1, 2, -3, 5, -8) \\
 r_6 = 13
 \end{array} \\
 \xleftarrow{\text{val}} \mathcal{B}([0, 1, 1, 1, 1, 1]) \xleftarrow{\Psi} \mathcal{J}(\frac{5}{8}) \xleftarrow{\Phi} \mathcal{M}(\frac{5}{8}) \\
 \begin{array}{l}
 \xrightarrow{\Psi} \begin{bmatrix} 5 \\ 8 \end{bmatrix}_q \\
 \xrightarrow{\Phi} q^{-1} \frac{q^5 + 2q^4 + q^3 + q^2}{q^4 + 2q^3 + 2q^2 + 2q + 1}
 \end{array}
 \end{array}$$

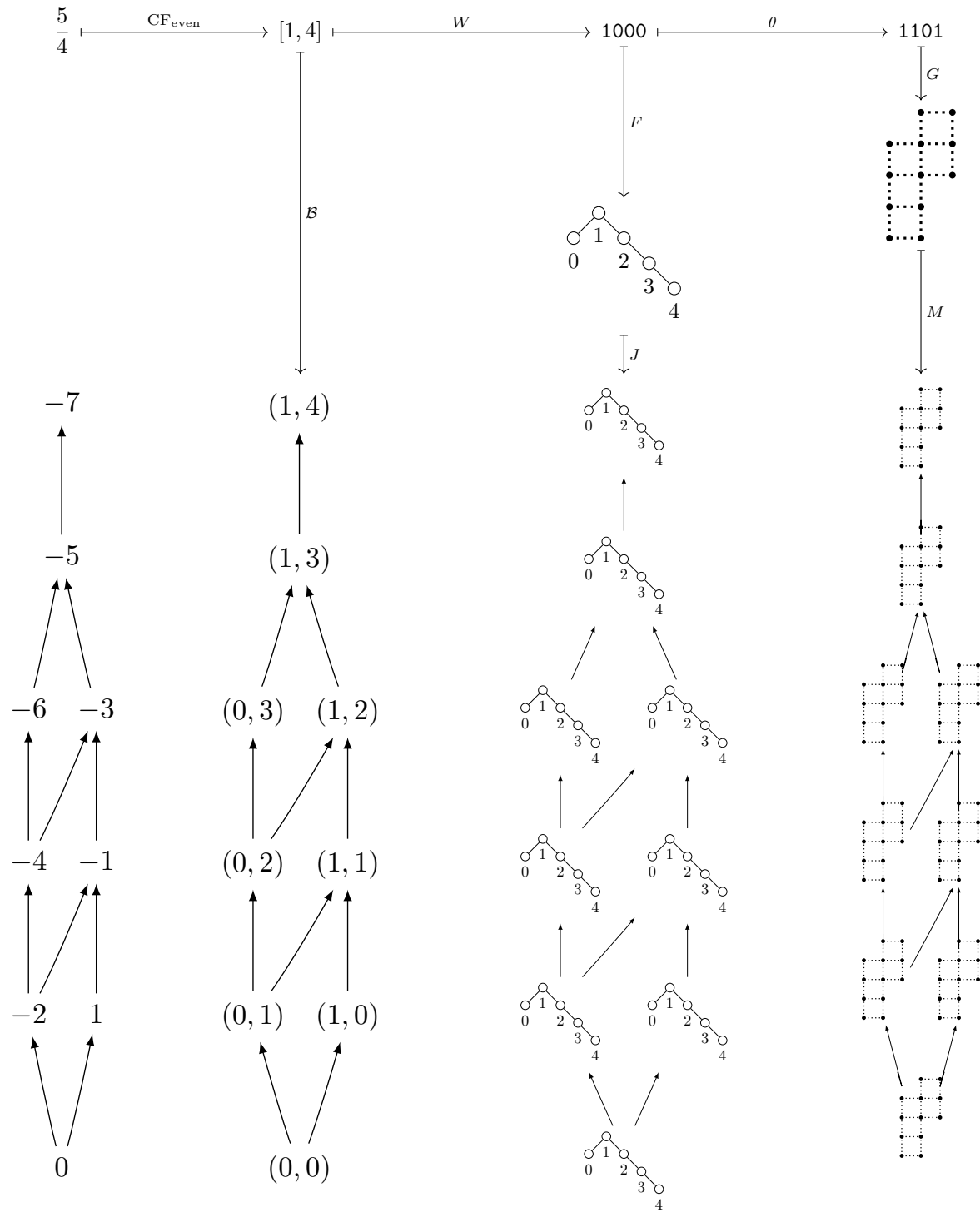


$$\begin{array}{c}
 \mathcal{Z}([0, 1, 2, 2]) \\
 = \mathbb{Z} \cap [-7, 5] \\
 \leftarrow \text{val} \longrightarrow \mathcal{B}([0, 1, 2, 2]) \xrightarrow{\Psi} \mathcal{J}(\frac{5}{7}) \xrightarrow{\Phi} \mathcal{M}(\frac{5}{7}) \\
 \begin{array}{l}
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -5) \\
 r_4 = 12
 \end{array} \\
 \begin{array}{l}
 \left[\begin{array}{c} 5 \\ 7 \end{array} \right]_q = q^{-1} \frac{q^5 + q^4 + 2q^3 + q^2}{q^4 + q^3 + 2q^2 + 2q + 1}
 \end{array}
 \end{array}$$



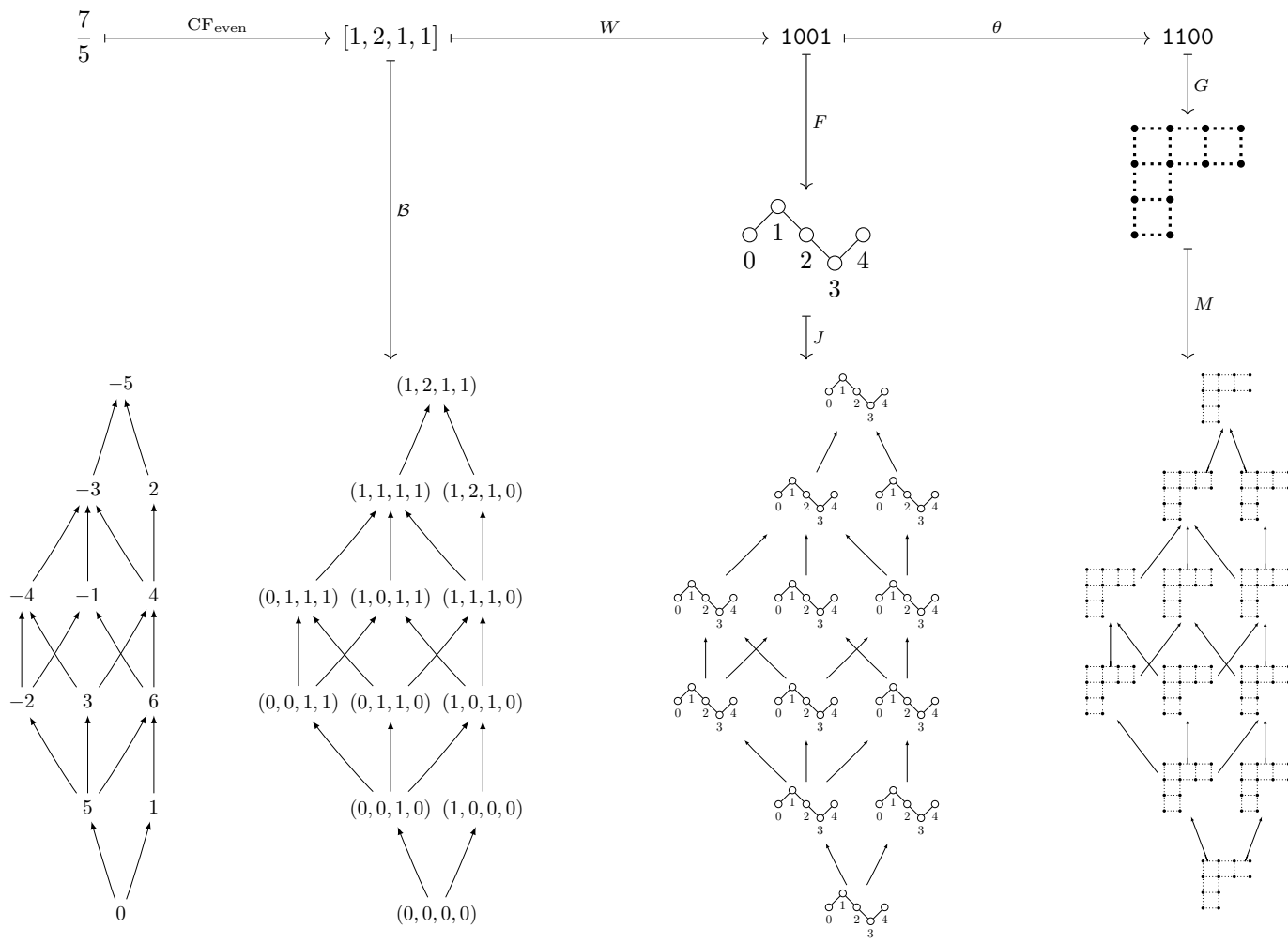
$$\begin{array}{c}
 \mathcal{Z}([0, 1, 3, 1]) \\
 = \mathbb{Z} \cap [-2, 7) \\
 (r_0, -r_1, r_2, -r_3) = (1, -1, 2, -7) \\
 r_4 = 9
 \end{array}
 \xleftarrow{\text{val}}
 \mathcal{B}([0, 1, 3, 1])
 \xleftarrow{\Psi}
 \mathcal{J}\left(\frac{4}{5}\right)
 \xleftarrow{\Phi}
 \mathcal{M}\left(\frac{4}{5}\right)$$

$$\left[\frac{4}{5} \right]_q = q^{-1} \frac{q^5 + q^4 + q^3 + q^2}{q^4 + q^3 + q^2 + q + 1}$$

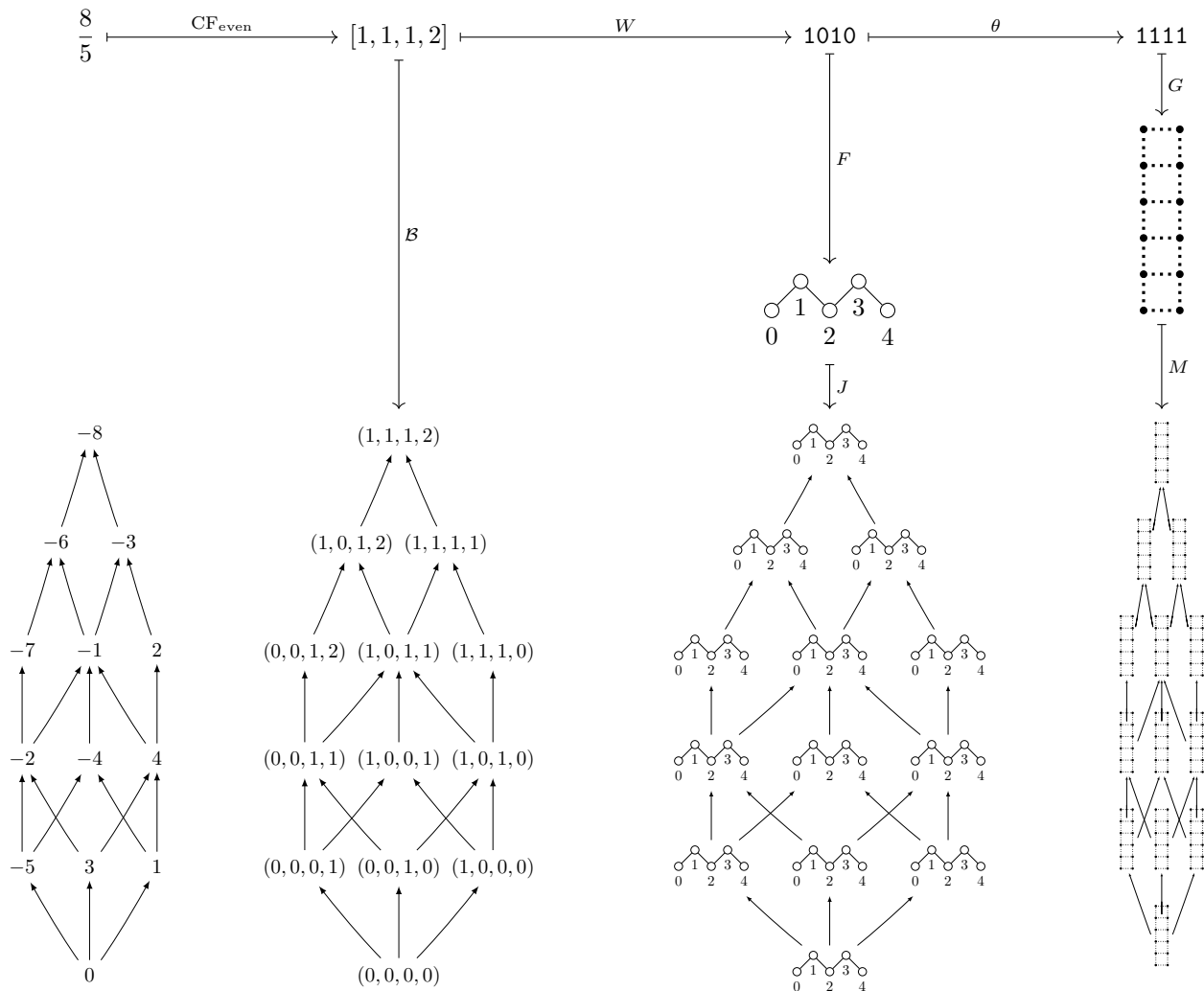


$$\begin{array}{c}
 \mathcal{Z}([1, 4]) \\
 = \mathbb{Z} \cap [-7, 2) \\
 (r_0, -r_1) = (1, -2) \\
 r_2 = 9
 \end{array}
 \xleftarrow{\text{val}}
 \mathcal{B}([1, 4])
 \xleftarrow{\Psi}
 \mathcal{J}\left(\frac{5}{4}\right)
 \xleftarrow{\Phi}
 \mathcal{M}\left(\frac{5}{4}\right)$$

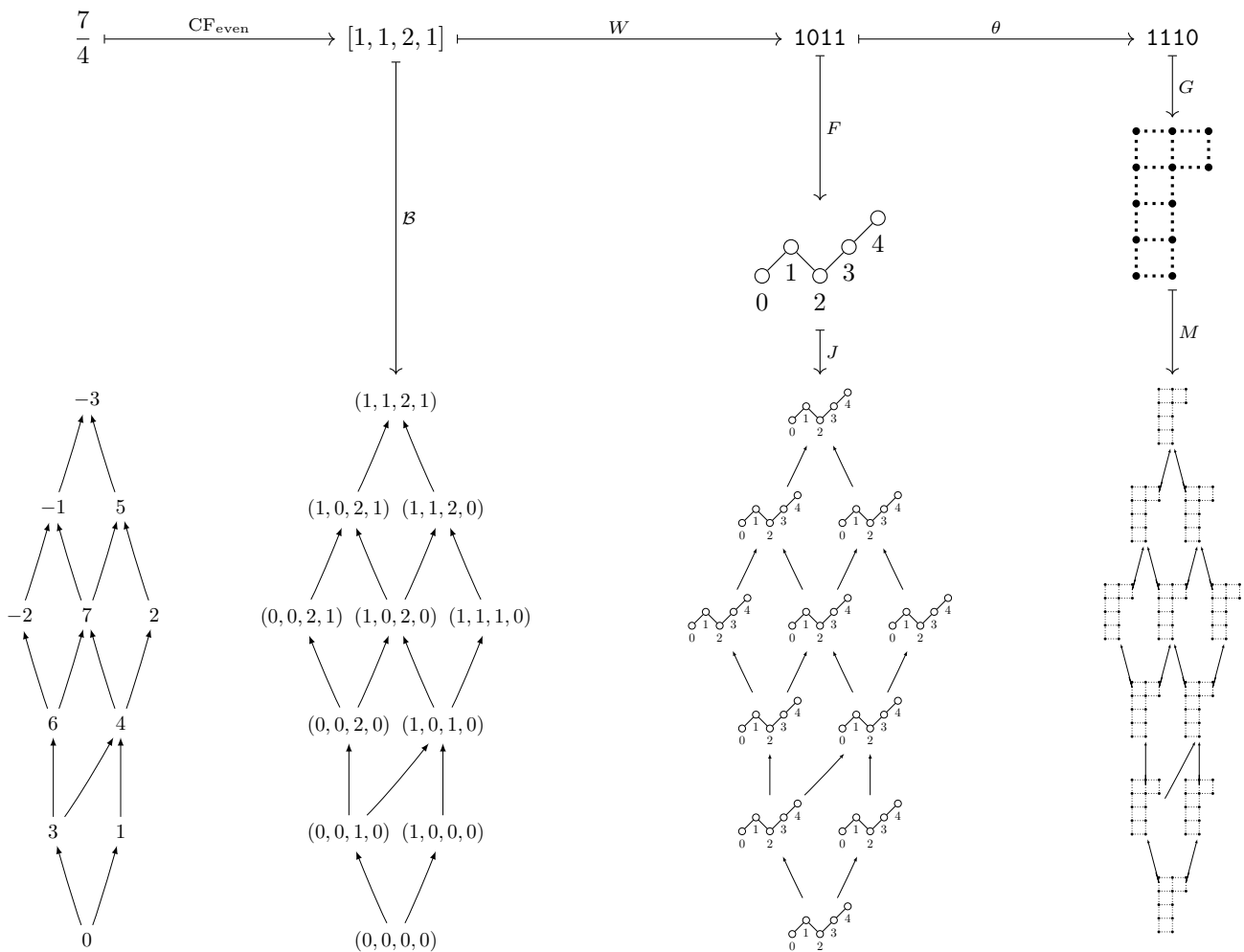
$$\left[\frac{5}{4} \right]_q = q^{-1} \frac{q^5 + q^4 + q^3 + q^2 + q}{q^3 + q^2 + q + 1}$$



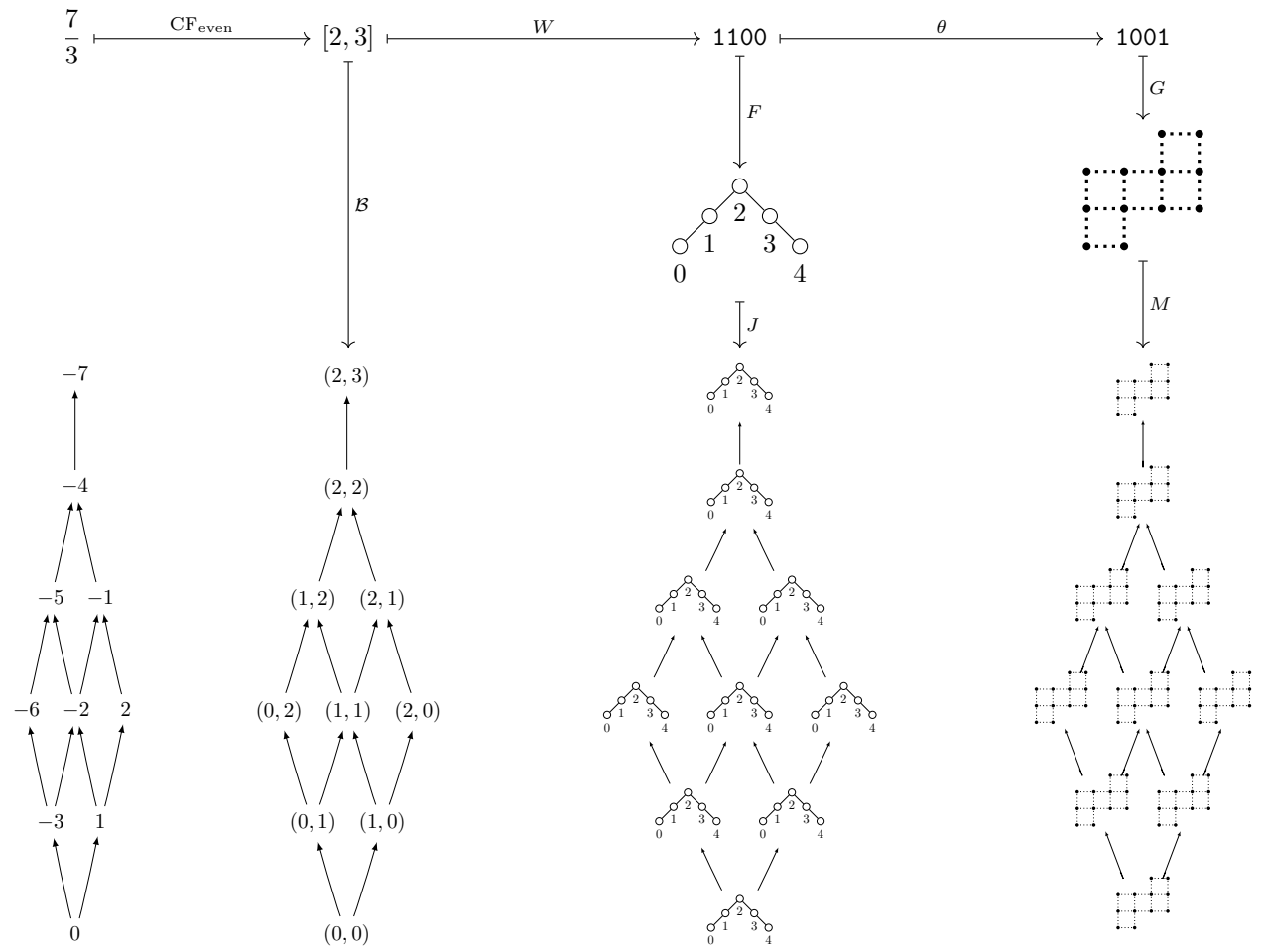
$$\begin{array}{c}
 \mathcal{Z}([1, 2, 1, 1]) \leftarrow \text{val} \mathcal{B}([1, 2, 1, 1]) \xleftarrow{\Psi} \mathcal{J}(\frac{7}{5}) \xleftarrow{\Phi} \mathcal{M}(\frac{7}{5}) \\
 = \mathbb{Z} \cap [-5, 7] \\
 (r_0, -r_1, r_2, -r_3) = (1, -2, 5, -7) \\
 r_4 = 12 \\
 \left[\frac{7}{5} \right]_q = q^{-1} \frac{q^5 + 2q^4 + 2q^3 + q^2 + q}{q^3 + 2q^2 + q + 1}
 \end{array}$$



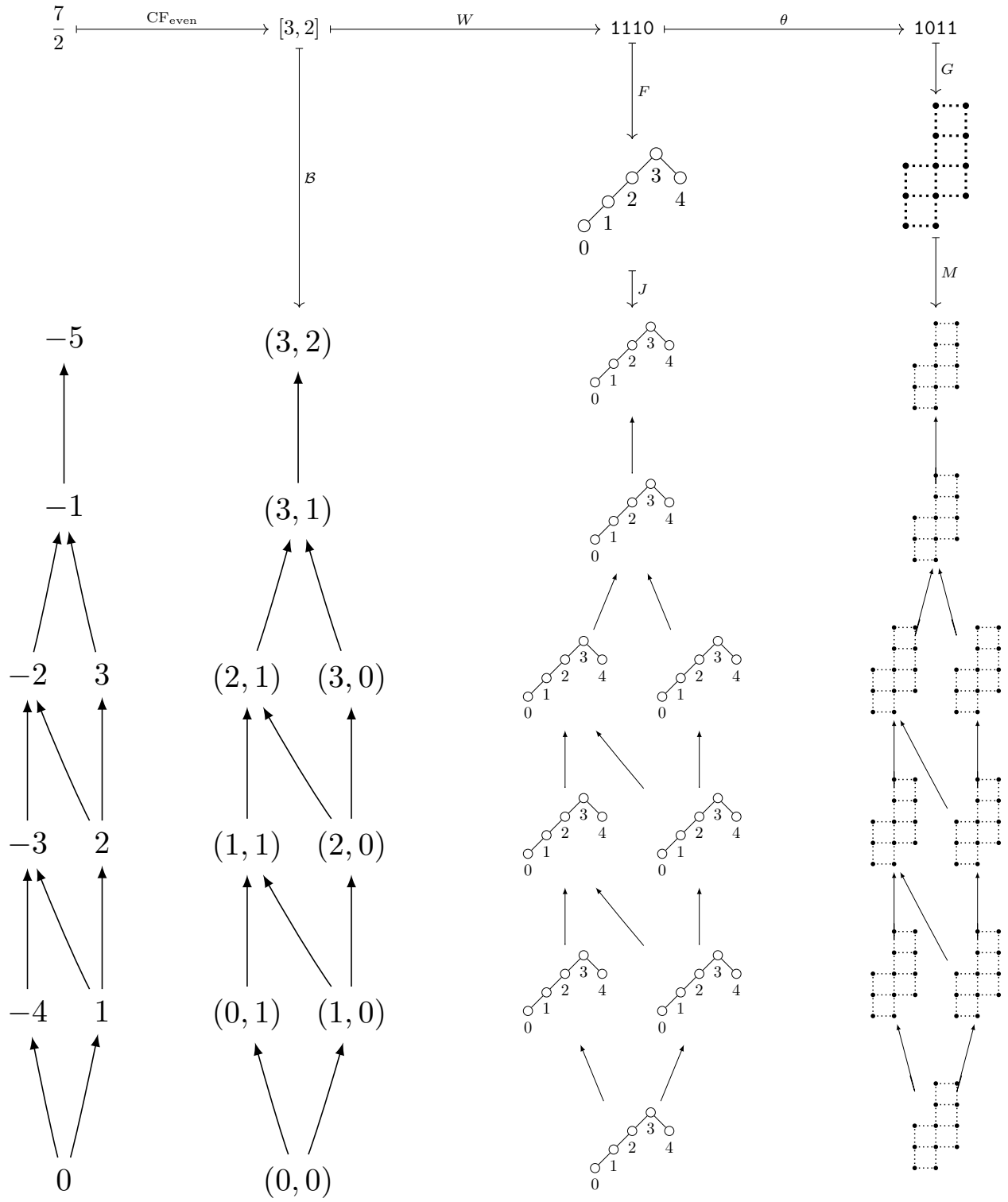
$$\begin{array}{c}
 \mathcal{Z}([1, 1, 1, 2]) \xleftarrow{\text{val}} \mathcal{B}([1, 1, 1, 2]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{8}{5}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{8}{5}\right) \\
 = \mathbb{Z} \cap [-8, 5) \\
 (r_0, -r_1, r_2, -r_3) = (1, -2, 3, -5) \\
 r_4 = 13 \\
 \left[\frac{8}{5} \right]_q = q^{-1} \frac{q^5 + 2q^4 + 2q^3 + 2q^2 + q}{q^3 + q^2 + 2q + 1}
 \end{array}$$



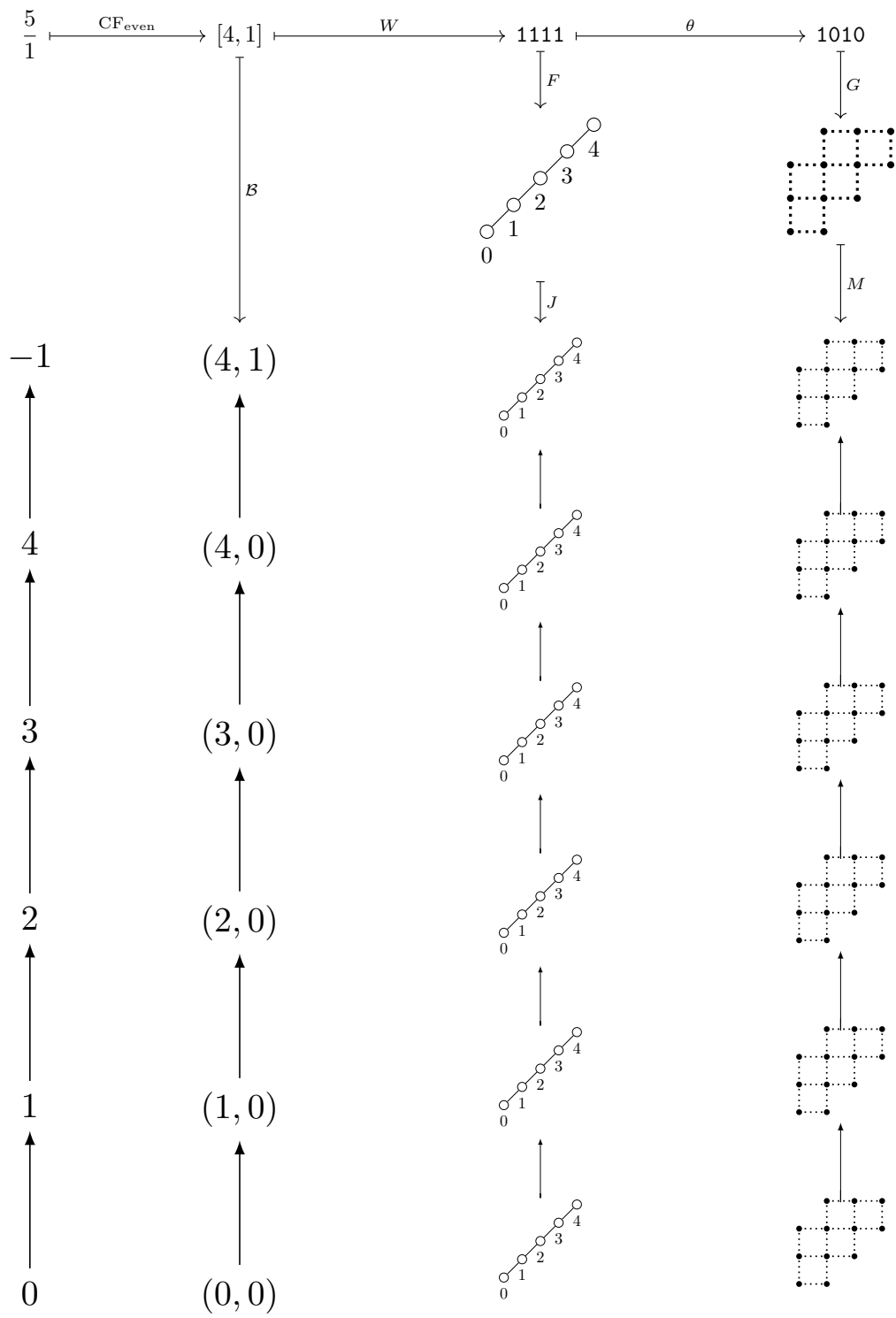
$$\begin{array}{c}
 \mathcal{Z}([1, 1, 2, 1]) \leftarrow \text{val} \mathcal{B}([1, 1, 2, 1]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{7}{4}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{7}{4}\right) \\
 = \mathbb{Z} \cap [-3, 8) \\
 (r_0, -r_1, r_2, -r_3) = (1, -2, 3, -8) \\
 r_4 = 11 \\
 \left[\begin{array}{c} 7 \\ 4 \end{array} \right]_q = q^{-1} \frac{q^5 + 2q^4 + 2q^3 + q^2 + q}{q^3 + q^2 + q + 1}
 \end{array}$$



$$\begin{array}{c}
 \mathcal{Z}([2, 3]) \\
 = \mathbb{Z} \cap [-7, 3] \\
 (r_0, -r_1) = (1, -3) \\
 r_2 = 10 \\
 \left[\begin{array}{c} 7 \\ 3 \end{array} \right]_q = q^{-1} \frac{q^5 + q^4 + 2q^3 + 2q^2 + q}{q^2 + q + 1}
 \end{array}
 \begin{array}{c}
 \xleftarrow{\text{val}} \mathcal{B}([2, 3]) \xleftarrow{\Psi} \mathcal{J}\left(\frac{7}{3}\right) \xleftarrow{\Phi} \mathcal{M}\left(\frac{7}{3}\right) \\
 \mathcal{B}([2, 3]) \rightarrow \mathcal{J}\left(\frac{7}{3}\right) \rightarrow \mathcal{M}\left(\frac{7}{3}\right)
 \end{array}$$



$$\begin{array}{ccccc}
 \mathcal{Z}([3, 2]) & \xleftarrow{\text{val}} & \mathcal{B}([3, 2]) & \xleftarrow{\Psi} & \mathcal{J}(\frac{7}{2}) & \xleftarrow{\Phi} & \mathcal{M}(\frac{7}{2}) \\
 = \mathbb{Z} \cap [-5, 4) & & & & & & \\
 (r_0, -r_1) = (1, -4) & & & & & & \\
 r_2 = 9 & & & & & & \\
 & & & & \downarrow & & \\
 & & & & \left[\frac{7}{2} \right]_q & = & q^{-1} \frac{q^5 + q^4 + 2q^3 + 2q^2 + q}{q + 1}
 \end{array}$$



$$\begin{array}{ccccc}
 \mathcal{Z}([4,1]) & \xleftarrow{\text{val}} & \mathcal{B}([4,1]) & \xleftarrow{\Psi} & \mathcal{J}\left(\frac{5}{1}\right) & \xleftarrow{\Phi} & \mathcal{M}\left(\frac{5}{1}\right) \\
 = \mathbb{Z} \cap [-1,5) & & & & \downarrow & & \downarrow \\
 (r_0, -r_1) = (1, -5) & & & & \left[\frac{5}{1} \right]_q & = & q^{-1} \frac{q^5 + q^4 + q^3 + q^2 + q}{1} \\
 r_2 = 6 & & & & & &
 \end{array}$$