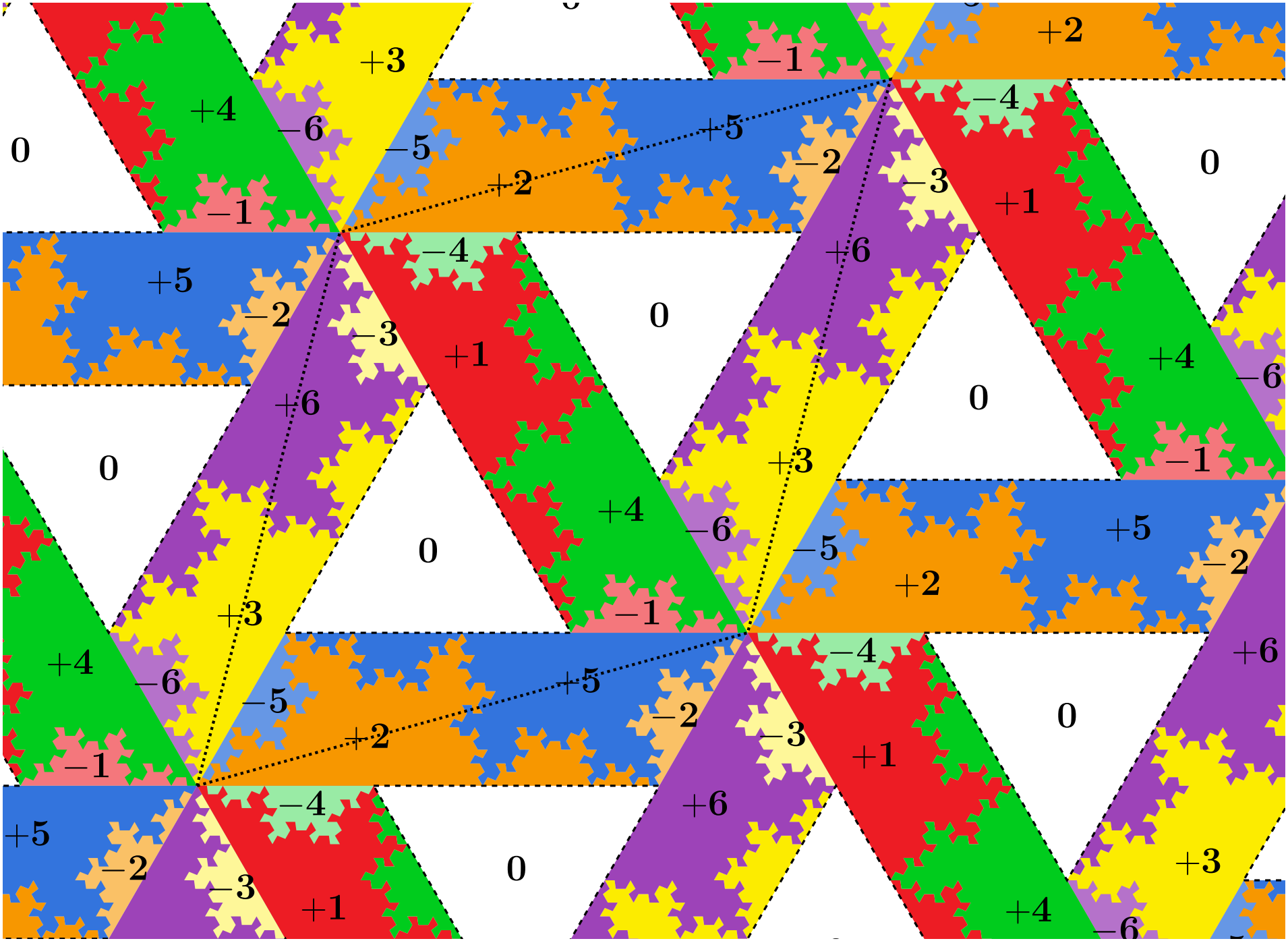
A construction of the hat tilings by a Markov partition

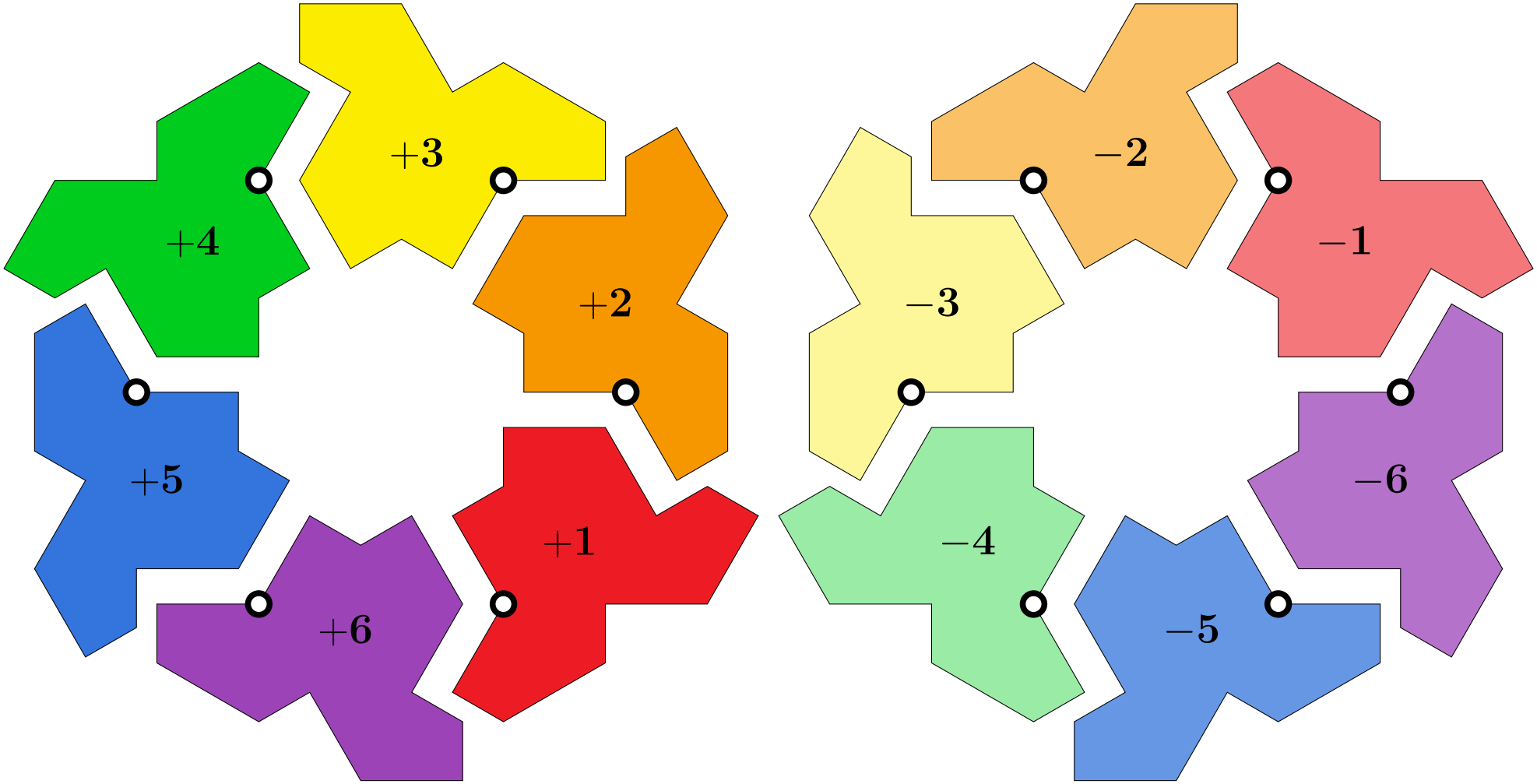
by ©Peter Selinger (https://www.mathstat.dal.ca/~selinger/) and ©Sébastien Labbé (http://www.slabbe.org/)

presented during the semester *Illustration as a mathematical research technique*, Institut Henri Poincaré, Paris, March 26th, 2026, arXiv:2604.20964

1 unit = $2\sqrt{3}$ cm



Selinger's choice for the anchors:



Key observation: in a tiling, these anchors live on the lattice $\mathbb{Z}[\omega]$ with $\omega = \exp(\pi i/3)$.
The anchors of two distinct tiles do not coincide.
Thus every tiling of the plane by copies of the hat can be described as a map

$$\mathbb{Z}[\omega] \to \{-6, -5, \ldots, 5, 6\}$$

where 0 means that *no tile is anchored at that vertex.*