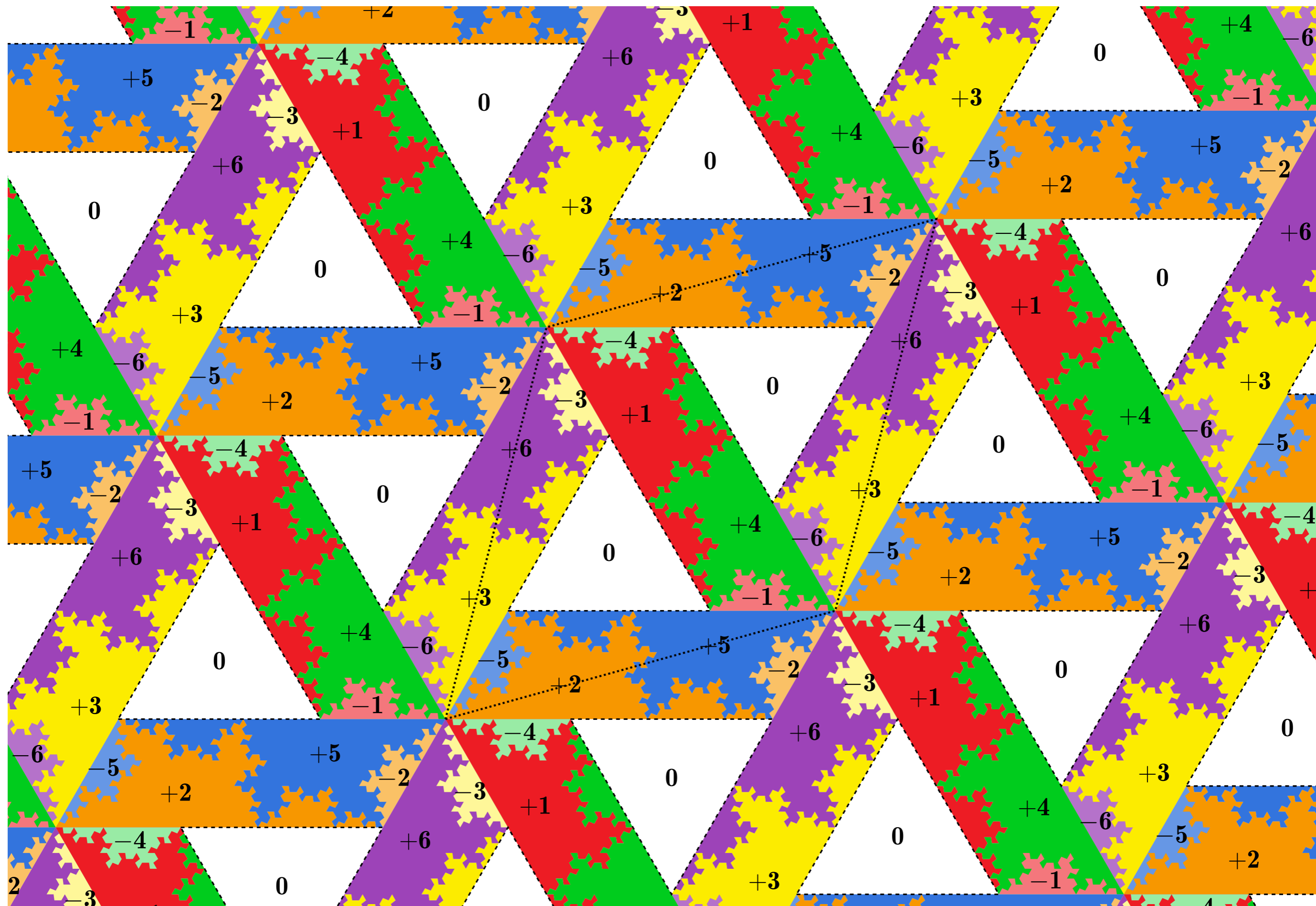


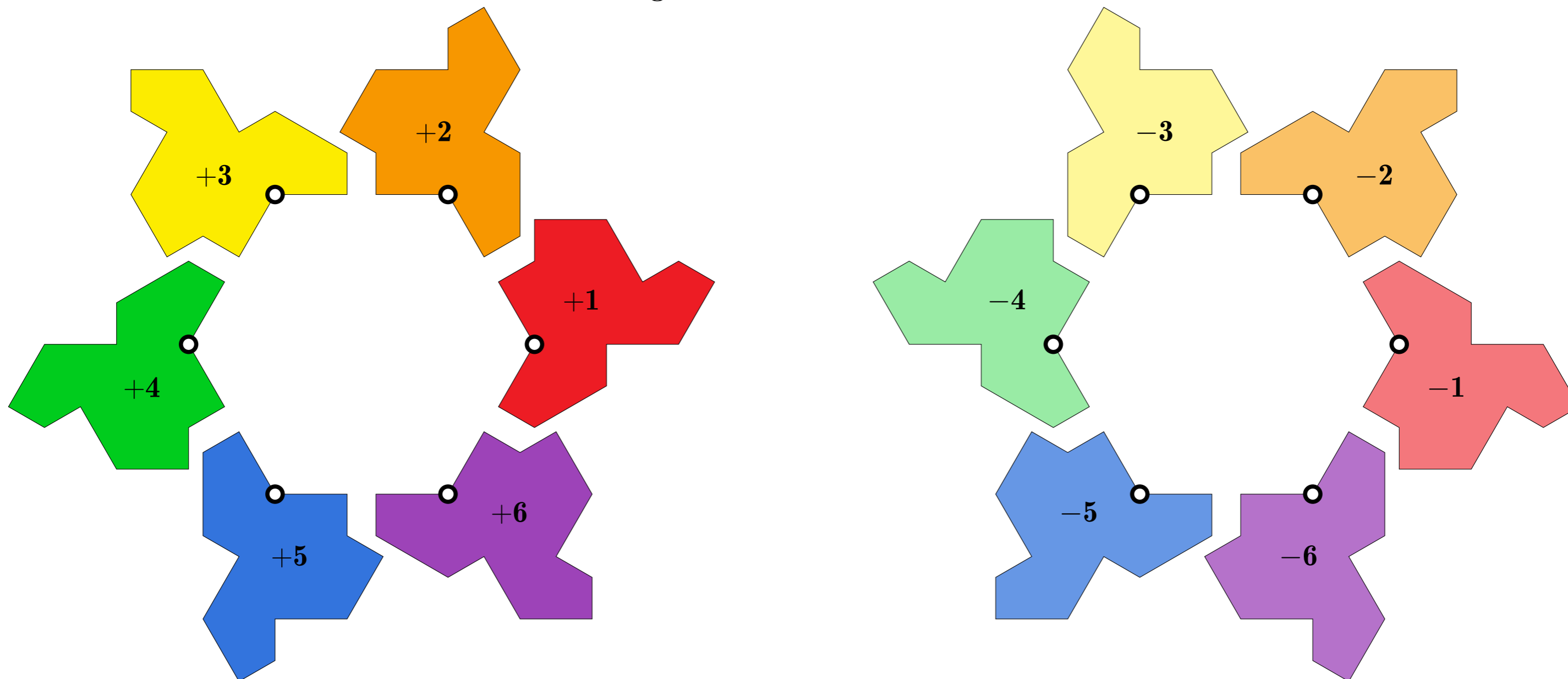
A CONSTRUCTION OF THE HAT TILINGS BY A MARKOV PARTITION

© by Peter Selinger (<https://www.mathstat.dal.ca/~selinger/>) and Sébastien Labbé (<http://www.slabbe.org/>)
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1 unit = $2\sqrt{3}$ cm



Selinger's choice for the anchors:



Key observation: in a tiling, these anchors live on the lattice $\mathbb{Z}[\xi]$ with $\xi = \exp(\pi i/3)$.

In a tiling, the anchors of distinct tiles do not coincide.

Thus every tiling of the plane by copies of the Hat can be described as a map

$$\mathbb{Z}[\xi] \rightarrow \{-6, -5, \dots, -1, 0, +1, \dots, +5, +6\}$$

where 0 means that *no tile is anchored at that vertex*.