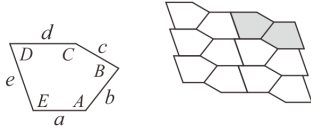


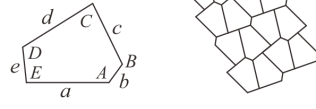
Type 1

$$A + B + C = 360^\circ.$$



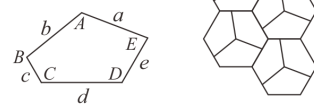
Type 2

$$A + B + D = 360^\circ, \\ a = d.$$



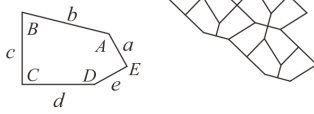
Type 3

$$A = C = D = 120^\circ, \\ a = b, d = c + e.$$



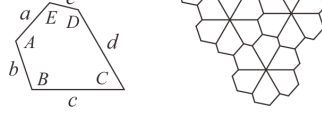
Type 4

$$C = E = 90^\circ, \\ a = e, c = d.$$



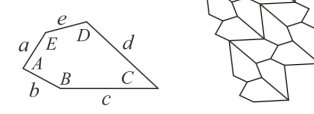
Type 5

$$A = 120^\circ, \\ C = 60^\circ, \\ a = b, c = d.$$



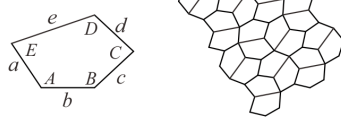
Type 6

$$A + B + D = 360^\circ, \\ A = 2C, \\ a = b = e, c = d.$$



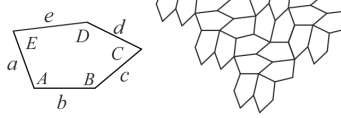
Type 7

$$2B + C = 360^\circ, \\ 2D + A = 360^\circ, \\ a = b = c = d.$$



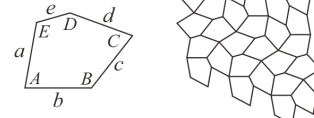
Type 8

$$2A + B = 360^\circ, \\ 2D + C = 360^\circ, \\ a = b = c = d.$$



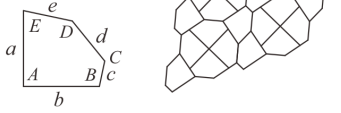
Type 9

$$2E + B = 360^\circ, \\ 2D + C = 360^\circ, \\ a = b = c = d.$$



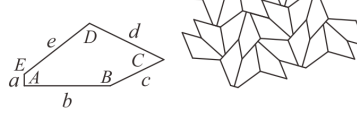
Type 10

$$A = 90^\circ, B + E = 180^\circ, \\ 2D + E = 360^\circ, \\ 2C + B = 360^\circ, \\ a = b = c + e.$$



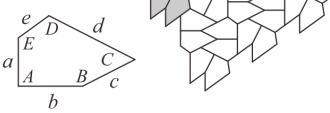
Type 11

$$A = 90^\circ, \\ C + E = 180^\circ, \\ 2B + C = 360^\circ, \\ d = e = 2a + c.$$



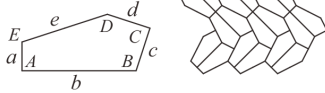
Type 12

$$A = 90^\circ, \\ C + E = 180^\circ, \\ 2B + C = 360^\circ, \\ 2a = d = c + e.$$



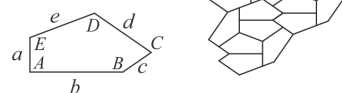
Type 13

$$A = C = 90^\circ, \\ 2B + D = 360^\circ, \\ B = E, \\ 2c = 2d = e.$$



Type 14

$$A = 90^\circ, \\ C + E = 180^\circ, \\ 2B + C = 360^\circ, \\ 2a = 2c = d = e.$$



Type 15

$$A = 90^\circ, B = 150^\circ, \\ C = 60^\circ, \\ 2a = 2b = 2d = c.$$

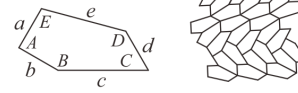


Figure. 1: Convex pentagonal tiles of 15 families. Each of the convex pentagonal tiles is defined by some conditions between the lengths of the edges and the magnitudes of the angles, but some degrees of freedom remain. For example, a convex pentagonal tile belonging to Type 1 satisfies that the sum of three consecutive angles is equal to 360° . This condition for Type 1 is expressed as $A + B + C = 360^\circ$ in this figure. The pentagonal tiles of Types 14 and 15 have one degree of freedom, that of size. For example, the value of C of the pentagonal tile of Type 14 is $\cos^{-1}((3\sqrt{57} - 17)/16) \approx 1.2099$ rad $\approx 69.32^\circ$. The pale gray pentagons in each tiling indicate a fundamental region (the unit that can generate a periodic tiling by translation only).

