# ON THE COMPUTATIONS MADE BY APERIODIC WANG TILES

SÉBASTIEN LABBÉ

## 1. Background on aperiodic Wang tilings

*Wang tiles* are unit square tiles with colored edges as in Figure 1. Given a finite set of Wang tiles, we consider *tilings* of the Euclidean plane using arbitrarily many copies of the tiles. Tiles are placed on the integer lattice points of the plane with their edges oriented horizontally and vertically. The tiles may not be rotated. The tiling is *valid* if every contiguous edges have the same color, see Figure 2.

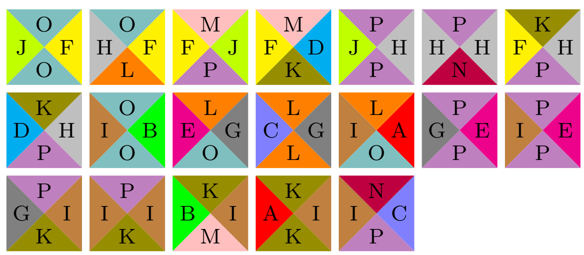


Figure 1. The set $\mathcal{U}$ of 19 Wang tiles introduced in [Lab18].

Given a finite set $\mathcal{T}$ of Wang tiles, we denote by $\Omega_\mathcal{T}$ the set of all valid tilings $f : \mathbb{Z}^2 \to \mathcal{T}$. The set $\Omega_\mathcal{T}$ is a 2-dimensional *subshift* as it is invariant under translations and closed under taking limits. Hence $\Omega_\mathcal{T}$ is called the *Wang shift* of $\mathcal{T}$. A nonempty Wang shift $\Omega_\mathcal{T}$ is *aperiodic* if none of the tilings in $\Omega_\mathcal{T}$ have a nontrivial period. Chapters 10 and 11 of [GS87] and the more recent book on aperiodic order [BG13] give an excellent overview of what is known on aperiodic tilings (in $\mathbb{Z}^d$ as in $\mathbb{R}^d$) together with their applications to physics and crystallography.

Examples of small aperiodic Wang tile sets include Ammann's 16 tiles [GS87, p. 595], Kari's 14 tiles [Kar96] and Culik's 13 tiles [Cul96]. The question of finding the smallest aperiodic set of Wang tiles was open until Jeandel and Rao proved [JR15] the existence of an aperiodic set of 11 Wang tiles and that no set of Wang tiles of cardinality $\leq 10$ is aperiodic.

## 2. Goal of the internship

The proof of aperiodicity for the set Kari's 14 tiles [Kar96] is very short (10 lines) and is based on arithmetic properties. The goal of the internship is to investigate the tile set shown in Figure 1 to figure out if there is a way to represent each Wang tile by some arithmetic equations like those satisfied by Kari's tile set. Such a interpretation would provide an explanation of aperiodicity in terms of the computations performed by those tiles.

---

*Date*: December 14, 2018.

*Key words and phrases.* Combinatorics and Wang tiles and tilings and aperiodic.

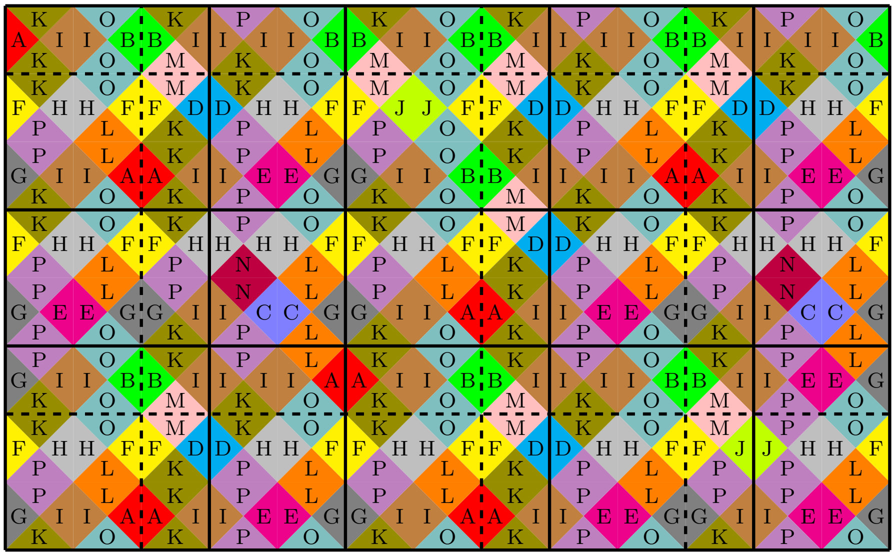

FIGURE 2. A $13 \times 8$ rectangle tiled by tiles from $\mathcal{U}$.

## References


[BG13] M. Baake and U. Grimm. *Aperiodic order. Vol. 1*, volume 149 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 2013.

[Cul96] K. Culik, II. An aperiodic set of 13 Wang tiles. *Discrete Math.*, 160(1-3):245–251, 1996.

[GS87] B. Grünbaum and G. C. Shephard. *Tilings and patterns*. W. H. Freeman and Company, New York, 1987.

[JR15] E. Jeandel and M. Rao. An aperiodic set of 11 Wang tiles. June 2015. `arxiv:1506.06492` .

[Kar96] J. Kari. A small aperiodic set of Wang tiles. *Discrete Math.*, 160(1-3):259–264, 1996.

[Lab18] S. Labbé. A self-similar aperiodic set of 19 Wang tiles. *Geom. Dedicata*, 2018. `https://doi.org/10.1007/s10711-018-0384-8`.



(S. Labbé) CNRS, LaBRI, UMR 5800, F-33400 Talence, France

*E-mail address*: `sebastien.labbe@labri.fr`

*URL*: `http://www.slabbe.org`