(1st) <u>A self-similar aperiodic set of 19 Wang tiles</u>, Geom Dedicata, Aug 2018

(2nd) <u>Subst. structure of Jeandel-Rao aperiodic tilings</u>, arxiv: 1808:07768

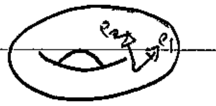
<u>Talk at Durham 7/09/2018</u>

Jeandel-Rao tiles: $T_0 = \left\{ \overset{\#0}{\boxed{2\,\overset{4}{\underset{1}{}}\,2}}, \overset{\#1}{\boxed{2\,\overset{2}{\underset{0}{}}\,2}}, \overset{\#2}{\boxed{3\,\overset{1}{\underset{1}{}}\,1}}, \ldots \overset{\#10}{} \right\}$

Wang shift: $\Omega_0 = \Omega_{T_0} = \{ \mathbb{Z}^2 \to T_0 \text{ valid Wang tilings} \}$

<u>Known</u> (JR, 2015) Ω_0 is aperiodic

<u>Motivation</u>:

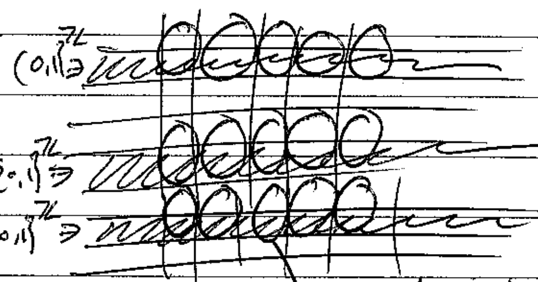


$\xhookrightarrow[\text{conjugacy}]{\text{meas.}} \Omega_0 \longrightarrow$ ~~meas. isomorphism~~

<u>Observations on T_0</u>:

	then $\notin\{0,1\}$	
then $\in\{0,1\}$	2 if $\in\{0,1\}$ 2	then $\in\{0,1\}$
	then $\notin\{0,1\}$	

which means that tiles 0,1 form non adjacent horizontal rows:



$\rightarrow$ Desubstitute!

$\boxed{\text{DEF}}$ T a Wang tileset, $\emptyset \neq M \subsetneq T$ is a <u>subset of markers in the direction e_2</u> if

$$M \odot^2 M, \quad M \odot^1 (T\setminus M), \quad (T\setminus M) \odot^1 M$$

$\rightarrow$ (Charlier, Kärki, Rigo)

are forbidden in Ω_T.

$\boxed{\text{EX}}$ $\{0,1\} \subset T_0$ is a subset of markers in direction e_2 for JR tiles.

<u>More observations on T_0</u>:

then $=9$
if $=0$

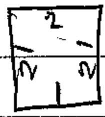
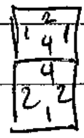
then $\in\{3,7,8,10\}$
if $=1$

$$\omega: \begin{cases} 0 \to \binom{9}{0}, \; 1 \to \binom{3}{1}, \; 2 \to \binom{7}{1}, \; 3 \to \binom{8}{1}, \; 4 \to \binom{10}{1} \\ 5 \to 2, \; 6 \to 3, \; 7 \to 4, \; 8 \to 5, \; 9 \to 6, \; 10 \to 7, \; 11 \to 8 \\ \cancel{12 \to 9}, \; 12 \to 10 \end{cases}$$

$\boxed{\text{THM}}$ If $\exists$ markers $M \subset T$ in the direction e_i, then $\exists$ Wang tile set S_L and a 2-dim morphism $\omega_L : \Omega_{S_L} \to \Omega_T$ s.t. $\omega_L(S_L) \subseteq (T\setminus M) \cup (M \odot^i (T\setminus M))$ which is <u>recognizable</u> and onto up to a shift.

$\hookrightarrow$ $\boxed{\text{DEF}}$ $\forall y \in \Omega_T$, $\exists$ at most one $k \in \mathbb{Z}^2$, $x \in \Omega_{S_L}$ s.t. $y = \sigma^k(\omega(x))$ with y_0 appears in the image of x_0

The substitution: $\omega_i \begin{cases} u \mapsto u \\ u \boxminus^i v \mapsto u \odot^i v \end{cases}$ or

 $= 0 \mapsto \begin{pmatrix} 9 \\ 0 \end{pmatrix} =$ 

Algorithms (in 2nd paper) : FINDMARKERS(T, i, r)
FINDSUBTITUTION(T, M, i, r)

• 2 useless tiles in T_4'
• No markers in T_4

$\Omega_0 \xleftarrow[\boxminus]{\omega_0} \Omega_1 \xleftarrow[\boxminus]{\omega_1} \Omega_2 \xleftarrow[\boxminus]{\omega_2} \Omega_3 \xleftarrow[\boxminus]{\omega_3} \Omega_4$

$\pi: \begin{smallmatrix} 0,6 \to 0 \\ 1,5 \to 1 \end{smallmatrix}\ \begin{smallmatrix} 2 \to 2 \\ 3 \to 3 \end{smallmatrix},\ 4 \to 4$ is an embedding!

Ω_5 • No markers in T_5

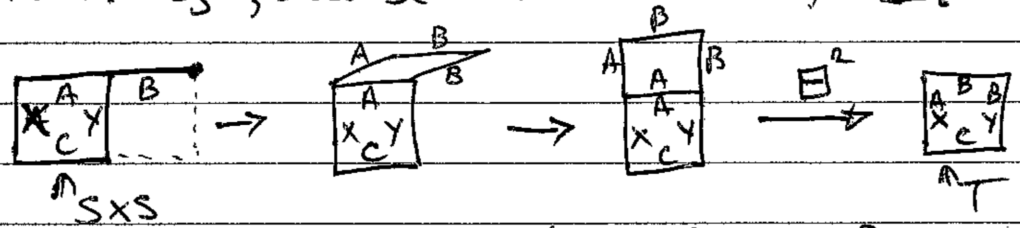
$\uparrow \eta$

$\Omega_6 \xleftarrow[\boxminus]{\omega_6} \Omega_7 \xleftarrow[\boxminus]{\omega_7} \Omega_8 \xleftarrow[\boxminus]{\omega_8} \Omega_9 \xleftarrow[\boxminus]{\omega_9} \Omega_{10} \xleftarrow[\boxminus]{\omega_{10}} \Omega_{11} \xleftarrow[\boxminus]{\omega_{11}} \Omega_{12} \cong \boxed{\Omega_u \xleftarrow[\boxminus]{\alpha} \Omega_v \xleftarrow[\boxminus]{\beta} \Omega_w \overset{\gamma}{\cong} \Omega_u}$
Paper #1
Ω_u is self similar, minimal, aperiodic

Sheer top. conjugacy $\eta: \Omega_6 \to \Omega_5$

• No markers in Ω_5, because markers are on lines of slope 1.

• Idea: for each domino



• leads to a top. conjugacy on tilings $\Omega_S \leftrightarrow \Omega_T$

Results

Thm Let $X_i = \overline{\omega_i(X_{i+1})}$ for $i = 0, \dots, 11$. $[\pi = \omega_4,\ \eta = \omega_5]$

• $X_{12} = \Omega_{12}$ is s.s. aperiodic minimal [1st paper]
• $X_i = \Omega_i$ is aper., minimal $\forall i,\ 5 \leq i \leq 11$.
• $X_i \subsetneq \Omega_i$ is ape. minimal $\forall i,\ 0 \leq i \leq 4$.
• Frequencies of tiles/patches in X_i $\forall i$.

Conjecture $\Omega_i \setminus X_i$ is of measure 0, $\forall i\ 0 \leq i \leq 4$.

$\Rightarrow X_0$ can be decomposed into patches of size 45, 70, 72, 112.