~~Complexité en facteurs des suites ARP~~

Complexité en palindromes des mots morphiques et conjecture classe P

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① Palindromes dans les mots

$A = \{a, b\}$ alphabet, $w = abaabb \in A^*$ un mot

$Pal(w) = \{\varepsilon, a, b, aba, aa, baab, bb\}$

$|Pal(w)| = 7 = 6+1 = |w|+1$

En général $|Pal(w)| \leq |w|+1$.

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p, q nouveaux palindromes

$\Rightarrow$ p pas nouveau ↯

Defect $D(w) = |w|+1 - |Pal(w)|$

② Palindromes ~~dans les~~ pt fixes de morphismes

- $\varphi : A^* \to A^*$ un morphisme
  $a \mapsto ab$
  $b \mapsto ba$

$x \in A^{\mathbb{N}}$ mot infini est un pt. fixe de φ si $x = \varphi(x)$

$x = \varphi(x)$
$= abba\,baab\,baababba\ldots$

$Pal(x) = \{\varepsilon, a, b, bb, abba, bab, abba, \ldots \varphi^2(a), \varphi^4(a), \varphi^6(a), \ldots \varphi^{2k}(a), \ldots\}$

$|Pal(x)| = \infty$

$\varphi^2 : a \mapsto abba$
$b \mapsto baab$

~~x palindromique si $|Pal(x)| = \infty$~~

- $\varphi : a \mapsto abb$
  $b \mapsto ba$

$x = abbba.baba.abb.\ldots$

$Pal(x) =$

$|Pal(x)| = 23$

$\{$ ε — aa, bb — baab, abba — ... — baabbbaabbbaab ; b — aba, bbb — babab, abbba — babbbab, bbaabbbaabb ; a — bab — bbabb, ababa $\}$

ababa $\in L(x)$

mais $\varphi\{$ |ba|baba|b $\notin L(x)$

b b|b b

$\varphi\{$ α impossible

Def x palindromique si $\#Pal(x) = \infty$

③ Conjecture classe P

Def (Hof, Knill, Simon, 1995) φ classe P (noté $\varphi \in P$) $\Leftrightarrow$ ($\exists$ palindrome p) ($\forall \alpha \in A$) ($\exists$ palindrome q_α) t.q. $\varphi(\alpha) = p q_\alpha$.

Lemme (HKS, 95) Si $\varphi \in P$, et $x \in \varphi(x)$ est un mot infini
$\Rightarrow x$ palindromique.

Preuve $w = w_1 w_2 \cdots w_n$ palindrome
$\Rightarrow \varphi(w) p = p q_{w_1} p q_{w_2} \cdots p q_{w_n} p$ est un palindrome. $\square$

Question (HKS, 95) "Clearly we could include into class P subst. of the form $s(b) = q b p$. We do not know whether all palindromic xs arise from subst. that are in this extended class P."

Restatement #1 $x = \varphi(x)$, x palindromic $\Leftrightarrow \varphi \in P$

False because $\varphi : \begin{array}{l} a \mapsto ab \\ b \mapsto ba \end{array} \notin P$ mais $x = \varphi(x)$ palindromic et $\varphi^2 \in P$

because $\varphi : \begin{array}{l} a \mapsto abba \\ b \mapsto bcab \\ c \mapsto cdd \\ d \mapsto dc \end{array} \notin P$ mais $x = \varphi(x) = abbc\ldots$ palindromic
φ n'est pas primitif.

Def φ primitif $\Leftrightarrow \exists k$ t.q. $\forall \alpha$ $\varphi^k(\alpha)$ contient toutes les lettres de A

Restatement #2 φ primitif, $x = \varphi(x)$
x palindromic $\Leftrightarrow \exists \varphi' \in \mathrm{Stab}(x) \setminus \{Id\}$ t.q. $\varphi' \in P$

Def $\mathrm{Stab}(x) = \{ \varphi : A^* \to A^* \mid \varphi(x) = x \}$

False because (Blondin-Massé, 08) $\varphi : \begin{cases} a \mapsto abbab \\ b \mapsto abb \end{cases}$ $x = \varphi(x)$

$\mathrm{Stab}(x) = \langle \varphi \rangle = \{\varphi^k\}$ (x rigide)
$\forall k$, $\varphi^k \notin P$ et x palindromique.
mais φ possède un conjugué $\varphi' \in P$.
$\varphi' : \begin{cases} a \mapsto bb.aba \\ b \mapsto bb.a \end{cases}$ $\qquad \varphi(w) a = a \varphi'(w) \quad \forall w \in A^*$

Restatement #3 φ primitif, $x = \varphi(x)$
"Class P Conjecture" x palindromic $\Leftrightarrow \exists \varphi' \in \mathrm{Stab}(x) \setminus \{Id\}$ t.q.
φ' has a conjugate in class P
noted $\varphi' \in P$

(4) Known Results

Prop (ABCD, 03) Solved Class P conjecture for periodic word x

Thm (Tan, 07) $\varphi : A^* \to A^*$ primitive, $|A| = 2$, $x = \varphi(x)$
x palindromic $\Leftrightarrow$ φ^2 has a conjugate in class P

Thm (Labbé, 08) φ primitive uniform, $A = \{a, b\}$, $x = \varphi(x)$
x palindromic $\Leftrightarrow$ $\varphi \in P$ or $\tilde{\varphi} \in P$ or $\varphi^2(a)$ and $\varphi^2(b)$ are palindromes.

Def φ uniform if $|\varphi(\alpha)| = |\varphi(\beta)|$ $\forall \alpha, \beta \in A$

Def $\hat{\varphi} : \hat{A} \to \hat{A} : a \mapsto \widehat{\varphi(a)}$

(5) Counter example to Class P conjecture

Thm (L, 13) $\exists$ primitive morphism $\gamma : A^* \to A^*$, $|A| = 3$, $x = \gamma(x)$ palindromic but none of $\varphi \in \mathrm{Stab}(x) \setminus \{Id\}$ has a conjugate in class P.

$$\gamma : \begin{cases} a \mapsto aca \\ b \mapsto cab \\ c \mapsto b \end{cases} \qquad x_\gamma = acabacacabacabacab\ldots$$

Lemme x_γ is palindromic

Preuve $p_0 = b$, $p_{k+1} = \gamma(cap_k)$ $k > 0$
$p_1 = \gamma(cab) = bacacab$
$p_2 = \gamma(bacacab) = bacacabacabacabacacab$
p_k palindrome $\forall k$ $\square$

Remarque $p_{k+1} = \underline{baca}\,\gamma(p_k)$
$\hookrightarrow$ not a palindrome

Remarque $x = \gamma(x_\gamma) = \pi(\underbrace{\mu(w)}_{\text{fixed point = purely morphic word}})$ morphic word

$$\pi : \begin{cases} x \mapsto ac \\ y \mapsto ab \end{cases} \qquad \mu : \begin{cases} x \mapsto xy \\ y \mapsto xxy \end{cases} \quad \text{et } \pi, \mu \in P$$

⑨ New statement

Restatement #4 φ primitive, $x = \varphi(x)$

x palindromic $\Leftrightarrow \exists f, g \in P'$ (conjugate to class P)
s.t. $x = f(u)$ and $u = g(u)$

Theorem (Harju, Vesti, Zamboni, arxiv 1311.0185, Nov 2013)

φ primitive, $x = \tau(y)$, $y = \varphi(y)$

x palindromic with finite defect $\Longrightarrow$ $\exists f, g \in P'$ s.t. $x = f(u)$ and $u = g(u)$
and x is rich (defect $= 0$)

This proves statement #4 for sequences x with finite defect

Def defect of infinite word u is $D(u) = \sup\{D(p) : p \text{ prefix of } u\}$

Finite Defect Conjecture (BBGL, 08)

φ primitif, $x = \varphi(x)$
$0 < D(x) < \infty \Rightarrow x$ periodic

(E. Vaslet, Bucci, 2012) counter example:
$$\begin{cases} a \mapsto aabcacba \\ b \mapsto aa \\ c \mapsto a \end{cases}$$

still open for injective morphism or for the binary alphabet