

Complexité en facteurs des suites ARP

Complexité en palindromes des mots morphiques et conjecture classique

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① Palindromes dans les mots

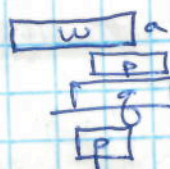
~~$A = \{a, b\}$ alphabet~~, $w = \underline{abaabb} \in A^*$ un mot

$$\text{Pal}(w) = \{\epsilon, a, b, aba, aa, baab, bb\}$$

$$|\text{Pal}(w)| = 7 = 6 + 1 = |w| + 1$$

En général $|\text{Pal}(w)| \leq |w| + 1$.

$$\text{Defect } D(w) = |w| + 1 - |\text{Pal}(w)|$$



$$|\text{Pal}(w)| \leq |w| + 1$$

p, q nouveaux palindromes

$\Rightarrow p$ pas nouveau

② Palindromes dans les pt fixe de morphismes

- $\varphi: A^* \rightarrow A^*$ un morphisme
 $a \mapsto ab$
 $b \mapsto ba$

$$x = \varphi(x) \\ = abba baabbaabba \dots$$

$$\text{Pal}(x) = \{\epsilon, a, b, bb, abba, babb, abba, \dots, \varphi^{2k}(a), \varphi^{2k}(b), \dots\}$$

$$|\text{Pal}(x)| = \infty$$

$$\varphi^2: a \mapsto abba, b \mapsto baab$$

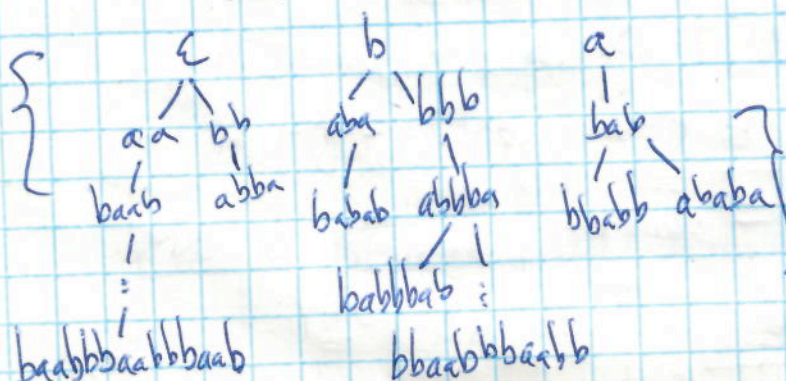
~~palindromes~~ si $|\text{Pal}(x)| = \infty$

- $\varphi^3: a \mapsto abb, b \mapsto baa$

$$x = abba, baba, abba, \dots$$

$$|\text{Pal}(x)| = 23$$

$$\text{Pal}(x) =$$



mais $ababa \in L(x)$
 $babababab \notin L(x)$

$\varphi \nmid b b b b$
 $\varphi \nmid a$ impossible

Def x palindromique si $\# \text{Pal}(x) = \infty$

③ Conjecture classe P

Def (Hofmann, Simon, 1993)

φ classe P $\Leftrightarrow \begin{pmatrix} \exists \text{ palindrome } P \\ (\exists \text{ palindrome } g) \text{ t.g. } \varphi(x) = P g x \end{pmatrix} (\forall x \in A^*)$

Lemme (HKS, 95) Si $\varphi \in P$, et $x \in \varphi(x)$ est un mot infini $\Rightarrow x$ palindromique.

Preuve

$w = w_1 w_2 \dots w_n$ palindrome $\Rightarrow \varphi(w) P = P g w_1 P g w_2 \dots P g w_n P$ est un palindrome. $\square$

Question (HKS, 95) "clearly we could include into class P subst. of the form $s(b) = g b P$. we do not know whether all palindromic x 's arise from subst. that are in this extended class P."

Restatement #1 $x = \varphi(x)$, x palindromic $\Leftrightarrow \varphi \in P$

False because $\varphi: \begin{matrix} a \mapsto ab \\ b \mapsto ba \end{matrix} \notin P$ mais $x = \varphi(x)$ palindromic et $\varphi^2 \in P$

because $\varphi: \begin{matrix} a \mapsto abba \\ b \mapsto baab \\ c \mapsto cdcd \\ d \mapsto dc \end{matrix} \notin P$ mais $x = \varphi(x) = abba \dots$ palindromic φ n'est pas primitif.

Def φ primitif $\Leftrightarrow \exists k \neq 0 \forall x \varphi^k(x)$ contient toutes les lettres de A

Restatement #2 φ primitif, $x = \varphi(x)$ x palindromic $\Leftrightarrow \exists \varphi' \in \text{Stab}(x) \setminus \{\text{Id}\}$ t.g. $\varphi' \in P$

Def $\text{Stab}(x) = \{ \varphi: A^* \rightarrow A^* \mid \varphi(x) = x \}$

False because (Baudin, Masser, 08) $\varphi: \begin{cases} a \mapsto abbaab \\ b \mapsto abba \end{cases} \quad x = \varphi(x)$

$\text{Stab}(x) = \langle \varphi \rangle = \{ \varphi^k \}$ (x rigide)

$\forall k, \varphi^k \notin P$ et x palindromique.

mais φ possède un conjugué $\varphi' \in P$.

$\varphi': \begin{cases} a \mapsto \text{abbaab} \\ b \mapsto \text{abba} \end{cases}$

$\varphi(w)a = a\varphi'(w) \quad \forall w \in A^*$

Restatement #3 φ primitif, $x = \varphi(x)$

x palindromic $\Leftrightarrow \exists \varphi' \in \text{Stab}(x) \setminus \{\text{Id}\}$ t.g. φ' has a conjugate in class P $\hookrightarrow$ noted $\varphi' \in P$

"Class P conjecture"

④ Known Results

Prop (ABCD, 03) Solved Class! conjecture for periodic word x

Thm (Tan, 07) $\varphi: A^* \rightarrow A^*$ primitive, $|A|=2$, $x = \varphi(x)$
 x palindromic $\Rightarrow \varphi^2$ has a conjugate in class P

Thm (Labbé, 08) φ primitive uniform, $A = \{a, b\}$, $x = \varphi(x)$
 x palindromic $\Rightarrow \varphi \in P$ or $\tilde{\varphi} \in P$ or $\varphi^2(a)$ and $\varphi^2(b)$ are palindromes.

Def φ uniform if $|\varphi(x)| = |\varphi(y)| \forall x, y \in A$

Def $\tilde{\varphi}: \tilde{A} \rightarrow \tilde{A} : a \mapsto \tilde{\varphi}(a)$

⑤ Counter example to Class! conjecture

Thm (L, 13) $\exists$ primitive morphism $\gamma: A^* \rightarrow A^*$, $|A|=3$, $x = \gamma(x)$ palindromic
 but none of $\varphi \in \text{Stab}(x) \setminus \{\text{Id}\}$ has a conjugate in class P .

$$\gamma: \begin{cases} a \mapsto aca \\ b \mapsto cab \\ c \mapsto b \end{cases}$$

$$x_\gamma = acabacacabacabacab \dots$$

Lemma x_γ is palindromic

Proof

$$p_0 = b, \quad p_{k+1} = \gamma(ca p_k) \quad k \geq 0$$

$$p_1 = \gamma(cab) = b a c a c a b$$

$$p_2 = \gamma(bacacab) = bacacabacabacabacacab$$

$$p_k \text{ palindromic } \forall k$$

□

Remark

$$p_{k+1} = \underline{baca} \gamma(p_k)$$

$\rightarrow$ not a palindromic morphic word

Remark

$$x = \gamma(x_\gamma) = \pi(\underbrace{\mu(w)}_{\text{fixed point} = \text{palindromic morphic word}})$$

$$\pi: \begin{cases} x \mapsto aca \\ y \mapsto ab \end{cases}$$

$$\mu: \begin{cases} x \mapsto xy \\ y \mapsto xxy \end{cases} \quad \text{et } \pi, \mu \in P$$

③ New statement

Restatement #4 φ primitive, $x = \varphi(x)$

x palindromic $\Leftrightarrow \exists f, g \in P^1$ (conjugate to class P)
s.t. $x = f(u)$ and $u = g(u)$

Theorem (Hajó, Vesti, Zamboni, arxiv/1311.0185, Nov 2013)

φ primitive, $x = \tau(y)$, $y = \varphi(y)$

x palindromic with finite defect $\Rightarrow \exists f, g \in P^1$ s.t. $x = f(u)$ and $u = g(u)$
and x is rich (defect = 0)

This proves statement #4 for sequences x with finite defect

Def defect of infinite word u is $D(u) = \sup \{D(p) : p \text{ prefix of } u\}$

Finite Defect Conjecture (BBGL, 08)

φ primitive, $x = \varphi(x)$

$0 < D(x) < \infty \Rightarrow x$ periodic

(E. Varlet, Bucci, 2012) counter example:

$\begin{cases} a \mapsto aabcbacba \\ b \mapsto aa \\ c \mapsto a \end{cases}$

still open for injective morphism or
for the binary alphabet

